The development and application of a mathematical model of enhanced oil recovery

by HARVEY S. PRICE
INTERCOMP Resource Development and Engineering, Inc.
Houston, Texas

INTRODUCTION

A great deal of attention has recently been focused upon "enhanced oil recovery"; that is, the application of recently developed processes for recovering oil that would normally remain in a reservoir following depletion by conventional techniques. Since this heretofore unrecoverable remaining oil amounts to about 300 billion barrels in the U.S. alone (see Figure 1), one can see why enhanced recovery is getting so much attention.

Among the broad class of enhanced oil recovery processes are processes called chemical flooding. These chemical flooding processes are extremely complex and really not completely understood. For example, a chemist in the laboratory can design a process which will recover 80 to 100 percent of the oil-in-place from a representative piece of reservoir rock; however, when these same systems are applied in the field, recoveries have ranged from 15 to 65 percent. This implies that there are interactions which sometimes occur in the reservoir that are sufficiently off design relative to laboratory work, to result in near or total process failure. To this technical uncertainty, add the problems of selecting a specific process for a given reservoir and the tremendous economic uncertainties that still exist; we see why chemical flooding is not projected to recover substantial amounts of oil for this country by the year 1985.

One way to accelerate the amount of oil recovery from chemical flooding is to obtain a better understanding of what takes place in a chemical flooding project. This understanding can be achieved through the interaction between mathematical models, laboratory results and actual field experiments. Much work is going on in the laboratory, and much work is going on in the field. However, until very recently there did not exist a mathematical model which could be used to better evaluate and understand the complex physics involved in all these processes and be used to scale the laboratory results to the field. INTERCOMP has developed such a chemical flooding model, and the development and application of this model is the subject of this paper.

The term "chemical flooding" used here is only one of many names used by the oil industry to describe similar oil recovery processes. Other names include micellar-polymer flooding, low tension waterflooding and soluble oil flooding, as well as several trademarked names for processes developed by specific companies. These have in common the injection of a chemical, normally a surfactant (or soap) to reduce the forces that trap oil in the rock in the presence of water. With these forces reduced, the oil can be mobilized and driven to a production well. Although the process is simple in concept, the specific interactions of the fluids involved are quite complex.

In order to model the behavior of chemical flooding, a number of primitive unknowns need to be identified and the complex partial differential equations which describe mass balances on these unknowns need to be solved in one, two and! three dimensions. This, however, is not all of the problem. Because of the important physical processes which are taking place, the mathematical solution of these equations needs to be far more accurate in some instances than in any other petroleum recovery technique which is currently being routinely modeled. This paper presents results of some approximations which have been successfully used by INTERCOMP to generate solutions with acceptable accuracy.

This model is currently being used in an application to help engineer a micellar-polymer flood in the Bell Creek field for Gary Operating Company. The model has been effectively used to characterize two quite different chemical flooding processes and has aided Gary in selecting the process which will be used in the field. A discussion of the selection process and some preliminary engineering results which should enhance the performance of Gary's field pilot are also presented. While the discussion of the Bell Creek Project is very preliminary, we believe that this is the beginning of an understanding which will ultimately lead to much better agreement between project design and field performance. If this goal is achieved, we will be well on our way to knowing how many of the 300 billion barrels of now unrecoverable oil can be recovered economically.
Chemical flooding is the name we shall use to denote these reservoir processes characterized by the injection of an agent to reduce the surface tension between oil and water and, hence, allow the displacement and recovery of oil that is normally trapped by capillary forces as a waterflood residual. The process is known alternatively throughout the industry as miscible-type waterflooding,1 micellar flooding,2,3 low tension waterflooding,4 surfactant flooding,5-7 and soluble oil flooding.8

Although chemical flooding could be applied for secondary as well as a tertiary oil recovery, the typical chemical flooding candidate is currently a watered-out oil zone; the in situ water being a normal oil field brine, high in total dissolved solids and divalent cations. The chemical flooding process thus consists of (1) a brine preflush to condition the formation and provide a controlled fluid environment that will allow optimum activity of the following surfactant system; (2) a small slug of a surfactant fluid, generally consisting of a dilute concentration of petroleum sulfonate in brine, with the possible additions of cosurfactants, polymers and other chemicals to stabilize the system, enhance the surfactant activity, reduce adsorption losses and control slug mobility; (3) a mobility control slug consisting of a dilute solution of polymer in brine, used to protect against backside dilution or overrunning of the surfactant slug by drivewater and to enhance areal and vertical sweep efficiency; and (4) a waterdrive to sweep the displaced oil, water and injected fluids to the producers.

As pointed out by Knight et al.,15 use of a brine preflush is not universal and certainly there are nuances that are unique to each company’s process. However, the above description appears to be the generally accepted characterization of the chemical flooding process.

The fluid characteristics and transport mechanisms important to the chemical flooding process have been studied extensively throughout the industry. A few of these studies are described in References 1-14 and we have included in our model the mechanisms and fluid properties indicated to be significant.

The chemical flooding system can be represented by six species or components flowing in a maximum of two phases. We may think of species one through six as water, oil, polymer, surfactant, salt and divalent ions, and denote the phases as aqueous and hydrocarbon. However, if we write the conservation equations for species three through six as to allow for (a) partitioning between aqueous and hydrocarbon phases, (b) adsorption on the reservoir rock, and (c) bulk transport and dispersion in both phases, other systems may be examined as well. In fact, the formulation used should allow the study of all of the mechanisms which have been indicated by the literature to be important.

Model formulation

The chemical flooding model solves for six components (or species) in two fluid phases. All six components may partition amongst the aqueous and hydrocarbon phases. Four components may disperse by both molecular diffusion and mechanical dispersion in both the longitudinal and transverse direction and in both phases. The same four components may adsorb on the reservoir rock, satisfying equilibrium sorption isotherms.

The primitive unknowns to be solved for by the simulator are:

- WA1 Mass fraction of water in the aqueous phase
- WH2 Mass fraction of oil in the hydrocarbon phase
- WA3 Mass fraction of component three (normally polymer) in the aqueous phase
- WA4 Mass fraction of component four (normally surfactant) in the aqueous phase
- WA5 Mass fraction of component five (normally monovalent salt) in the aqueous phase
- WA6 Mass fraction of component six (normally divalent salt) in the phase saturation
- SA Aqueous phase saturation
- p Aqueous phase pressure

The equations to be solved by the simulator are:

\[
\frac{\partial}{\partial t} (\phi S_a p_a WA1 + \phi S_b p_b SH1) + \nabla \cdot (p_a WA1 U_a + p_b WH1 U_b) - \bar{q}_1 = 0
\]  

\[
\frac{\partial}{\partial t} (\phi S_a p_a WA2 + \phi S_b p_b WH2) + \nabla \cdot (p_a WA2 U_a + p_b WH2 U_b) - \bar{q}_2 = 0
\]  

\[
\frac{\partial}{\partial t} (\phi S_a p_a WA3 + \phi S_b p_b WH3 + \phi \rho_a) + \nabla \cdot (p_a WA3 U_a + p_b WH3 U_b) - \nabla \cdot (\phi S_a p_a KS_a V WA3 + \phi S_b p_b KS_b V WH3) - \bar{q}_3 = 0
\]
Components 4 (surfactant), 5 (monovalent salt) and 6 (divalent salt) equations are the same as that for Component 3, except that \( \phi_{0} \) is replaced by \( \phi \).

There are three constraint equations for the system. The first requires the sum of saturations be equal to one. This equation is satisfied by setting \( S_{h} = 1 - S_{a} \) in a six component balance equations. The other two constraint equations require that the sum of the mass fractions in each phase equal one. These equations are left explicitly in the formation as Equations (7) and (8). By ordering the unknowns as shown on the list of primitive unknowns, Equations (7) and (8) eventually become the "saturation" and "pressure" equations.

Assuming an isothermal system with compositions independent of pressure, we have the following equilibrium relationships for partitioning of species one through six between the aqueous and hydrocarbon phases and for adsorption of species three through six on the reservoir rock:

\[
W_{Hi} = f_{i}(W_{Aj})_{j=1-6} \quad \text{for } i = 1 \text{ through } 6 \tag{9}
\]

\[
\rho_{h} = f_{ih}(W_{Aj})_{j=1-6} \quad \text{for } i = 3 \text{ through } 6 \tag{10}
\]

For equations of state, we have

\[
\rho_{h} = \rho_{h}(W_{Ai}, p_{h})_{i=1-6} \tag{11}
\]

\[
\rho_{h} = \rho_{h}(W_{Hi}, p_{h})_{i=1-6} \tag{12}
\]

\[
U_{h} = -\frac{k_{h}}{\mu_{h}} (\nabla p_{h} - \rho_{h} g \nabla D) \tag{13}
\]

\[
U_{h} = -\frac{k_{h}}{\mu_{h}} (\nabla p_{h} - \rho_{h} g \nabla D) \tag{14}
\]

where

\[
k_{a} = k_{a}(S_{a} W_{Ai})_{i=1-6} \tag{15}
\]

\[
k_{h} = k_{a}(S_{h} W_{Ai}, U_{h})_{i=1-6} \tag{16}
\]

\[
\mu_{a} = \mu_{a}(W_{Ai}, S_{a} U_{h})_{i=1-6} \tag{17}
\]

\[
\mu_{h} = \mu_{a}(W_{Ai}, S_{h})_{i=1-6} \tag{18}
\]

Assuming that the capillary pressure is a function of both saturation and composition, we have finally

\[
p_{h} = p_{a} + P_{e}(S_{a} W_{Ai})_{i=1-6} \tag{19}
\]

MODEL SOLUTION

As if developing numerical solutions to Equations (1) through (19) is not complex enough, the numerical solution needs to be extremely accurate in order to be able to model the phenomena of dispersion and fingering. If it weren’t for these special problems, the numerical solution to Equations (1) through (19) could follow any normal finite difference or finite element type approximation, and for this reason will not be addressed in this paper. We will, however, address our attention at the specific problems of dispersion and fingering since these are truly important physical phenomena to be modeled and help to make some of the interesting points of this paper.

Figure 2 is included here to illustrate the effects of numerical dispersion. This figure displays the results of a simple, one-dimensional calculation. If the solution were infinitely accurate, the high spike shown on this figure would result from the effects of actual physical dispersion on the solution. As you can also see on Figure 2, the very smeared curve displays the effects of large numerical truncation error which has the exact same effect as a large physical dispersion. Figure 3 is an illustration depicting the phenomenon of viscous fingering. This occurs whenever a more mobile fluid is injected into a less mobile fluid; the more mobile fluid wants to finger through the less mobile fluid to the producing well providing a thief zone for the injected fluid. This phenomenon results in cycling of the injected fluid without really displacing the fluid in place. In chemical flooding, because the costs of the chemicals injected are so extremely high, all of the processes are designed around putting in as little chemical in the ground as is absolutely necessary.
Because of this, the effects of both physical dispersion and viscous fingering are extremely important in the field and need to be modeled accurately if realistic results are to be obtained.

There are two approaches to solving partial differential equations describing the chemical flooding process and still be able to achieve the kind of accuracies required in order to model dispersion and fingering. The first approach is to use high order finite element approximations and the second approach is to use some local grid modification such that the accuracy can be concentrated in the areas where it is required and coarse grids can be used elsewhere. The high order finite element approximations are depicted by Figure 4 where I have illustrated here third order, cubic Hermite polynomials which can very effectively be used to generate fourth order accurate approximations. Figures 5, 6 and 7 illustrate a specific type of local grid refinement. Here we have an original triangle grid in Figure 5 and two levels of refinement shown right at the injection well which is in the lower left-hand corner of Figure 6. Figure 7 illustrates a later point in time when this refinement has moved in somewhat away from the injection well in order to accurately describe a front which is moving away from the injection well. This is a local refinement that needs to be time varying and move along with the front.

To illustrate the importance of these phenomena, I've shown on Figure 8 the effects of dispersion on recovery. You will note that as the physical dispersion is reduced, the peak concentration of the injected slug is increased and, therefore, the amount of oil recovered by continuous displacement of the injected fluid is significantly increased. Figure 9 is an illustration of a computer plot which shows a first order approximation to the injection of a slug of displacing fluid at a one-to-one mobility ratio. You will note how dispersed this particular slug is relative to Figure 10, which represents a fourth order correct solution to this problem. You will note also Figure 11 where the use of local grid refinement has been used how much sharper and higher the
peak slug concentrations are than in Figure 9. Figure 12 is an illustration of a fourth order finite element approximation's ability to actually model viscous fingering in an unfavorable mobility ratio displacement and Figure 13 is an illustration of how the model is able to accurately predict recoveries at low dispersion levels for unfavorable mobility ratio displacements by introducing nominal heterogeneities into the system to propagate viscous fingers.

One further phenomenon which results when finite difference approximations are used to model unfavorable mobility ratio displacements is that of grid orientation. Figure 14
shows a repeated five-spot pattern and illustrates the two different grids that could be used to represent the flow of fluids in a five-spot. You will note that if the regions are totally homogeneous and if all the injection wells have the same rates and all the producing wells have the same rates, each of these repeated patterns should perform identical to
every other repeated pattern. You will note from this figure that one of the grids is parallel to the coordinate axes while the other grid is running at an angle of 45° to the coordinate axes. These two grids are called parallel and diagonal for the sake of discussion. You will note in Figure 15 how the recovery performance for a very simple displacement process can vary depending on whether one uses the parallel or the diagonal grid. As mentioned before, these results should be identical and yet there is as much as a 20 percent difference in the recovery of oil after one pore volume of fluid injected. This is clearly an intolerable error and needs to be eliminated. It can also be seen on this same curve that high order finite element approximations do not exhibit this grid orientation phenomenon. In fact, none of the finite element approximations exhibit grid orientation effects which plague finite difference schemes.

We have illustrated in this section how the problem of describing a physical process such as chemical flooding requires numerical approximations which not only solve complex partial differential equation with complex nonlinearities, but which also must describe the important physics of the problem. Finite element approximations are definitely recommended when high order accuracy to model effects such as physical dispersion and/or viscous fingering are required.

MODEL APPLICATION

Once a complex physical model, such as the one described by Equations (1) through (19), has been coded and debugged and running smoothly on a large-scale digital computer, one of the first things that needs to be done is to validate the model against actual physical processes in order to determine that the model does in fact describe the complex physics that it was intended to describe. This is really the stage of model development in which INTERCOMP is currently engaged; however, we have modeled two complex chemical flooding schemes and will illustrate here how the model was utilized in these particular instances.

The actual study that was performed was to model two competing design philosophies and to utilize the model to determine which of these should be implemented in Gary Operating Company’s Bell Creek micellar-polymer pilot. The two chemical design philosophies which were modeled were (1) a small slug high concentration “soluble oil” system, and (2) a large slug dilute concentration, “optimal salinity” design.

A number of laboratory experiments were run in order to obtain basic data for the model as well as to test the design philosophies in displacing oil from the Brea core. While a large number of experiments were actually modeled, I have chosen just two here to illustrate the actual capabilities of the chemical flood program. You will note from Figure 16 how good an actual match was obtained for a .09 pore volume slug using the high water content “HWC” process and from Figure 17 we have illustrated the match for the soluble oil process using a .03 pore volume slug. The interesting features about these history matches were that both
model runs utilized basically the same data and that only the different description of the physics resulted in the different model predictions. While these history matches are not perfect, they could easily have been further adjusted and fine tuned. However, it was not felt that this was necessary since the purpose here was really to demonstrate the model's capability to describe the physics of the two completely different systems. Once the history match was obtained then the model was used to predict the ultimate recovery from the cores using various weight percent surfactant concentrations. You see from Figure 18 that here again the model did an excellent job of reproducing laboratory experiments and that clearly the soluble oil system, at least in the core, was significantly better than the high water content process.

Once this history match was obtained the model was then utilized to make both areal and vertical cross section runs to see how these processes might deteriorate in the field. It is interesting to note that once we made these runs and combined the areal and the vertical runs to estimate what would be recovered from the pilot that the model predicted a 55 percent recovery with a soluble oil process and only a 35 percent recovery with a high water content process. These results are still preliminary since we did not at this point look at any degradation due to layering for these systems. However, it is very interesting to note that a process which recovers almost 90 percent of the oil in a core would be predicted to recover only 55 percent of the oil in an idealized pattern. This is far more like what is actually being observed in the field and is extremely encouraging that we have now reached the point where mathematical models can be used to predict oil recovery by chemical flooding.

Again while this work is still very preliminary, I would like to point out one other very interesting thing that has been learned to date from this particular study. The field pilot has been designed as a five-spot with this expanded to the full field originally if the pilot is successful. However one of the interesting things that was noted in this work is that because of the diverging nature of the flow in a five-spot as the slug moves away from the injection well very
little oil is displaced around the edges of the pattern. In fact, almost no oil is being mobilized around the edges and most of the oil being recovered from the pilot is coming right out of the stream tubes that run between the injector and the producer. This gives the strong indication that line drives or patterns other than five-spots might be considerably better for recovering oil via chemical flooding. This work will be expanded upon and may in fact even result in some design modification for this very exciting and interesting pilot project.

CONCLUSIONS

In this paper we have presented some of the interesting aspects in the development and application of a chemical flood model. This clearly illustrates how computers and mathematical models can play an extremely important role in aiding the oil industry in recovering additional oil. The complex model which was developed is now being utilized to better understand laboratory work which will in turn enable us to scale the very important laboratory experiments to actual field scale projects and hopefully enable us to obtain a better understanding why some processes seem to work and other processes seem to fail. Once a process is characterized and the reservoir is characterized we feel that a mathematical model of this sort is an excellent predictive tool and should go a long way toward enabling the oil industry to recover a substantial fraction of this 300 billion barrels of oil which is left in the ground and currently called unrecoverable.

REFERENCES
