General equilibrium models for energy policy analysis

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INTRODUCTION

In this paper we shall provide an elementary overview of the concept of general economic equilibrium in the concrete context of the U.S. energy market. With this aim in mind, we shall introduce and discuss several important, large-scale energy models which embody the assumptions of the methodology of economic equilibrium. The models which will be discussed were all developed since the OPEC embargo of 1973—an event which has provided the impetus for development of sophisticated computer models useful in energy planning and policy-making.

In the first section, the models of Hudson and Jorgenson and Hnyilicza are introduced. The discussion here will hopefully provide the reader with a clear diagrammatic exposition of the concept of general economic equilibrium. By "economic equilibrium" we mean a state of the economy in which prices are such as to equate the supply and demand for all commodities under consideration. These two particular models provide a good starting point for our discussion since they are very broad in scope, encompassing both the energy and the non-energy sectors of the U.S. economy. In doing so they take account of the important linkages which exist between the energy and non-energy sectors.

Then in the second section, we introduce the SRI National energy model, sometimes referred to as the SRI-Gulf model since it was originally developed with the collaboration of the Gulf Oil Corporation. Our focus in this section is on the algorithm through which economic equilibrium is determined in a dynamic economic context. An understanding of the algorithm not only sponsors an in-depth understanding of the SRI model and "how it works," but permits an appreciation of the important role of computation in large-scale models. Finally, in the last section, we conclude with some remarks on the role of economic models in energy policy analysis.

THE HUDSON-JORGENSEN/HNYILICZA MODELS—AGGREGATE MODELS OF ENERGY AND ECONOMIC GROWTH

Background

In 1974, Edward Hudson and Dale Jorgenson introduced a model of energy and economic growth. The Hudson-Jorgenson model consisted of two somewhat separate submodels: a macro-economic model of economic growth, and a multi-sector interindustry model. The model was the first to make use of a new and advanced methodology for econometric estimation in a general economic equilibrium context. This methodology is known as "translog economic modeling" in the technical literature.

An ostensible problem with the Hudson-Jorgenson model was its failure to integrate its two submodels in a completely satisfactory way. It was not clear that the inputs and outputs of the two submodels were compatible. This difficulty provided the motivation for Esteban Hnyilicza to develop a model which, while very similar to the Hudson-Jorgenson model, differed from it in its treatment of economic growth. Specifically, Hnyilicza has attempted to build a model in which the multi-sector inter-industry model and the macro-economic growth model are satisfactorily integrated. The Hnyilicza model also differs from the Hudson-Jorgenson model in being much more highly aggregated. Specifically, it consists of only two sectors, namely the energy and non-energy sectors of the economy. In other important respects, such as use of the "translog" methodology, the two models are much the same.

In this section, we are going to discuss the Hnyilicza model at some length. The purpose of discussing this model rather than the Hudson-Jorgenson model is that our diagrammatic mode of exposition of the methodology used in both models requires that we limit ourselves to a simple model with very few sectors.

Introduction

Hnyilicza has constructed an integrated model for the purpose of analyzing the relationship between economic...
growth, and the (equilibrium) prices and quantities of both energy and non-energy goods. His model provides the basis for computing a trajectory of future market clearing (i.e., "equilibrium") prices and quantities of the following commodities:

1. Energy consumption goods
2. Non-energy consumption goods
3. Energy intermediate goods
4. Non-energy intermediate goods
5. Capital services to the energy sector
6. Capital services to the non-energy sector
7. Labor services
8. Energy imports
9. Non-energy imports

These outputs are generated on a period-by-period basis as the solution to a model consisting of fifty-five (nonlinear) simultaneous equations. By analyzing time-series projections of the prices and quantities of these goods, Hnyilicza can diagnose the relationship between economic growth on the one hand, and the relative prices of energy and non-energy goods on the other hand. It is also possible to study the rate of substitution of capital and labor for energy, as well as the economic impact of alternative tax and conservation policies.

Overview of the model

Figure 1 displays the overall structure of the Hnyilicza model. There are three basic model sectors: industrial production, consumption, and investment. The fact that these sectors are enclosed by solid as opposed to dashed lines is supposed to describe the fact that the outputs of these sectors are determined endogenously within the model. This is not the case with the government sector and the foreign sector. The latter two (which are enclosed by dashed lines) are largely exogenous to the model.

Before delving into the details of the three sectors, let us briefly summarize what takes place in the model in hopes of better motivating Figure 1 and in hopes of providing the reader with a guide to the model. One preliminary word of warning is in order. It is extremely difficult to describe what "goes on" in any general equilibrium economic model on a sector-by-sector basis. The reason for this is simply that what happens in any one sector affects every other sector. Nonetheless, we have tried to motivate the model by means of a sector-by-sector discussion since there seems to be no satisfactory expository alternative.

Two things happen in the consumption sector. First, the "national household's" inter-temporal utility function determines the split between present consumption and future consumption (i.e., savings). More specifically, the household is assumed to determine a utility-maximizing split between consumption and savings as an (econometrically estimated) function of wealth, the wage rate, and government transfer payment levels. Second, the consumer decides upon an intra-period split between leisure, energy consumption goods, non-energy consumption goods, and capital services. The optimal split is that which is utility-maximizing, subject to the consumer's budget constraint. This split will of course be a function of the relative prices of the various consumption goods, and the wage rate, among other things.

The two industrial production sectors—energy and non-energy—determine an optimal (profit-maximizing) level of outputs, and an optimal (profit-maximizing) configuration of inputs used in producing the equilibrium output levels.

Finally, the investment sector determines the level of capital services which will be available in any given period. As we shall see, this level depends on both the efficiency of capital, and on the level of capital stocks in the preceding period. The investment sector also determines the overall level of gross investment in any given period, as well as the fractions of gross investment going to the energy sector, the non-energy sector and the household.

A diagrammatic exposition

An important question arises concerning the best way in which to explain and motivate a model which is as general as the Hnyilicza model. Two approaches seem to be possible: a mathematical approach and a diagrammatic approach. We have settled upon a diagrammatic exposition of the model. More specifically, we shall motivate the model by discussing each of the three sectors (production, consumption, and investment) from the standpoint of the household, and investment (production, consumption, and investment) from the standpoint of the household.

Before superimposing the various supply and demand curves which we "extract" from each of the three sectors. The price-quantity pairs of all of the points of intersection of these supply and demand curves will clearly constitute a general economic equilibrium. And these equilibrium prices and quantities coincide with the values of the endogenous variables which are determined when the model is solved.

In light of our decision to explicate the model in terms of the supply and demand relationships that are implicit in the equilibrium equations of the model, a caveat must be issued from the outset. Whenever the reader sees a picture of a supply curve or a demand curve, he should realize that he is beholding an illusion. For in point of fact, there are no such things as supply and demand curves per se. There are only conditional supply and demand curves. Let us illustrate this point since it often engenders confusion. Consider a two
commodity world consisting of coffee and sugar. In general, the reader would be hard pressed to spell out his price-demand relationship for coffee. He would only be able to do so if he knew how much sugar he would be allotted. Thus we can speak of a demand curve for coffee conditional upon a particular consumption level of sugar. For this reason, it is somewhat artificial to characterize market equilibrium below in terms of a set of supply curves and demand curves.³

Methodology of the model

Industrial production sector

The function of the production sector in the context of the larger model is twofold:

a. to determine an equilibrium factor input mix which is optimal (profit maximizing) for each industry in producing its total output;
b. to determine the total (equilibrium) output levels that meet the demands of the intermediate market and of the final goods market.

As we saw in Figure 1 and as we see in more detail in Figure 2 on the next page, we can think of the production sector as an input-output structure. We summarize this input-output structure in Table I.

³ Nonetheless, it will be true that if we somehow knew the equilibrium quantities of all commodities, then we could sketch a set of conditional supply and demand curves whose points of intersection do constitute the market equilibrium. The problem of course is that one does not know the equilibrium in advance.

It is important to realize that even though we are speaking of production in “input-output” terms, the Hnyilicza model is not an input-output model in the classical sense of that term. Rather, it is a generalized input-output model. In such models the input-output (transactions) coefficients, rather than being given exogenously (as would be the case in the economy whose technologies permitted no substitutability among inputs), are functions of the prices of all inputs and the configuration of final demand. Specifically, we can write

\[ a_{ij} = f(w, p, y, x) \]

where:
- \( w \) is the vector of input factor prices
- \( p \) is the vector of output prices
- \( y \) is the vector of final demands
- \( x \) is the vector of gross outputs.

The equilibrium coefficients \( \{a_{ij}\} \) that are implicitly solved for by the model designate the profit-maximizing

| TABLE I |
| Inputs |
| Labor Services |
| Capital Services* |
| Energy Intermediate Goods |
| Non-Energy Intermediate Goods |
| Energy Consumer Goods |
| Energy Intermediate Goods |
| Energy Nonenergy Intermediate Goods |
| Energy Inputs |
| Energy Intermediate Goods |
| Energy Nonenergy Intermediate Goods |
| Labor Services |
| Capital Services |
| Energy Intermediate Goods |
| Non-Energy Intermediate Goods |

*Capital services are the annual services generated by existing stocks of capital goods. These services are provided to the productive process. "Capital goods" have a life exceeding one year, by definition, whereas "Intermediate goods" are consumed by the production process within a year.
number of units of commodity $i$ which are utilized in the production of each unit of commodity $j$. Let us now press towards a deeper understanding of what is going on in the production sector. A result of this will be an enhanced understanding of the equilibrium coefficients $\{a_{ij}\}$.

The most straightforward way in which to characterize what goes on in the production sector of the model is to depict the various supply curves and demand curves which are implicit in the equilibrium equations which characterize rational economic behavior on the part of the two industries: energy and non-energy.

The entrepreneur must make essentially two different types of decisions. He must decide upon a profit maximizing level of output of the commodities he manufactures; and he must decide upon a profit maximizing mix of inputs to be used in meeting his (optimal) production schedule.

Clearly his optimal choice of output levels and input mixes will be a function of the relative prices of outputs and inputs. Indeed, it is customary to speak of the relationship between the price of a given input and the quantity demanded by the entrepreneur at that price as the derived factor demand curve of the input (or "factor") in question. And the relationship between the optimal output level of a given output and the price of the output is summarized by the supply curve of the output.

With this in mind the reader is referred to Figure 2, where we identify the various derived demand curves and supply curves that are generated within the production sector. Let us summarize the information contained in the figure.

The derived factor demand curves and the supply curves

There is one derived factor demand curve for each input. Since both the energy and the non-energy sector use five inputs, there are therefore ten derived demand curves in all:

<table>
<thead>
<tr>
<th>Energy</th>
<th>Non-Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor Services</td>
<td>Labor Services</td>
</tr>
<tr>
<td>Capital Services</td>
<td>Capital Services</td>
</tr>
<tr>
<td>Energy Intermediate</td>
<td>Energy Intermediate</td>
</tr>
<tr>
<td>Goods</td>
<td>Goods</td>
</tr>
<tr>
<td>Imports</td>
<td>Imports</td>
</tr>
</tbody>
</table>

Demands:

The derived demand curves associated with these input demands are illustrated in Figure 2.

With respect to supply, the energy sector produces only two goods whereas the non-energy sector produces three
An input-output interpretation

We are now in a position to give an interpretation to the equilibrium Input-Output coefficients \( \{a_{ij}\} \). Note in Figure 2 that in two (and only in two) of the markets pictured there do we have well-defined supply and demand curves. This is the case in the markets for energy intermediate goods and non-energy intermediate goods. Suppose now that we could “complete” our description of the remaining eight markets by obtaining the missing supply and demand curves. (As will be seen, these missing curves are generated in the other two sectors of the model to which we shall turn shortly.) Suppose finally that we were to take note of the equilibrium point in these ten markets, that is, the equilibrium prices and quantities of the ten commodities.

It would then be straightforward to compute the equilibrium input-output coefficients either in dollar terms or in physical terms—using simple arithmetic. The collection of coefficients so obtained would coincide with the collection \( \{a_{ij}\} \). That is, these are the equilibrium input-output coefficients associated with profit-maximizing behavior on the part of the industries with respect to the choice of input mix and the choice of output level.

The consumption sector

Figure 3 portrays the consumption sector in some level of detail. Looking at the top part of the figure it is clear that two things take place in the consumption sector model. First, an “intertemporal” utility function determines a utility-maximizing split between present and future consumption. This utility function is an econometrically estimated function of:

- Time
- Interest rate
- Existing wealth
- Wage rate
- Government payments to household (e.g., welfare)
Second, an "intratemporal" utility function determines the split within a given period of household expenditures between the four consumption goods, namely energy consumption goods, non-energy consumption goods, capital services, and leisure. By capital services we mean the annualized value of the services which flow from consumer durables or residential stock. The intratemporal utility function which performs this split is an econometrically estimated function of price, the level of consumption of each good, and time.

A supply and demand curve interpretation

A deeper understanding of the consumption sector can be obtained by considering the supply and demand curves which are implicit in the equilibrium equations characterizing utility-maximizing behavior on the part of the household. These price-quantity relations are shown in the lower portion of Figure 3. Interpretation of all of these curves is straightforward with the exception of the supply curve of labor services. This curve is in fact the inverse of the demand curve for leisure services. As the wage rate increases, leisure becomes more expensive. Work is substituted for leisure, which increases the supply of labor.

Investment and capital accumulation

Formally speaking, the investment sector is part of the production sector. Nonetheless, it is useful to isolate it as a separate sector for the purposes of enhancing an understanding of capital accumulation in the Hnyilicza model. The situation is pictured in Figure 4. Consider the upper part of the figure. The supply of new investment goods that come on stream in a given year \( t \) appears on the left. Next, the supply of new investment goods is split three ways into the shares of investment goods going to the energy, non-energy, and domestic sectors. This split is realized by the use of a multiplier which reflects historical ratios in the apportionment of investment goods.

Moving further to the right, we add in the depreciated value of capital stock for the respective sectors. These are the values of stocks at the end of the previous year. The result of this addition is the current level of capital stock.

Moving yet further to the right in Figure 4, the values of the current level of capital stock in the two industrial sectors are multiplied by the efficiency of capital. The result of this operation is defined to be the level of capital services in period \( t \).

We exhibit the current supply of capital services generated in this fashion within each of the two industries as two supply curves. The reader will note that these supply curves are perfectly inelastic. The reason for this is that the level of capital services is not a function of any current prices. One might think that this level would be a function of the current price of new investment goods. This is not the case, for in this model there is a one period lag between the time when capital goods are produced and the time when they begin to generate capital services. The investment goods which come "on stream" in year \( k \) and which appear in the upper left of Figure 4 were actually produced in year \((k-1)\).
Two other price-quantity curves appear in the figure. First, there is a demand curve for investments goods. This demand curve is seen to be perfectly elastic. Moreover, it appears as a dashed line. Why? We have used a dashed line to indicate that this demand curve is an artificial construct.

For in the Hnyilicza model, the (equilibrium) level of investment goods is, in fact, "supply-determined." Whatever quantity of investments goods is supplied is assumed to be absorbed—irrespective of own price. It is for this latter reason that the curve appears as perfectly elastic.

In point of fact, it is possible to introduce a demand function for investment goods in the Hnyilicza model. For one of the basic equilibrium conditions of the model is that investment equals income minus consumption. [This equilibrium condition is imposed on the model.] Consumption, in turn, is a function of income and of the price of labor, capital, etc. Thus, the demand for investment goods is indeed a function of "prices" in the Hnyilicza model. Our point above is simply that it is not helpful to characterize a market of investment goods per se, and to plot the demand for investment goods as a function of the price for investment goods. For, as we said above, whatever quantity of investment goods is produced is assumed to be demanded.

Last of all, there is a demand curve for savings. This appears as a dashed vertical line. Why? The reason for this is that there is no demand curve* for savings per se in the model. Rather, savings is assumed to be equal to savings. Once again, just as in the case of the demand for investment goods above, it is possible to assert that there is a demand function for savings embedded in the Hnyilicza model. For the so-called "static equilibrium condition" of this type of growth model is that savings equals income minus "desired" consumption. This relationship is imposed on the model, and serves to relate industry demand for savings to consumption and hence to relative prices, since consumption depends in part on prices.

The reader who finds the rather arbitrary treatment of investment and savings perplexing should be aware of the motivation for the static equilibrium conditions such as "Savings=Investment" that are imposed on the model. The rationale for these assumptions is simply to insure consistency between the aggregate savings decisions by individuals on the one hand, and the aggregate investment decisions of producers on the other hand. There would be no need, in principle, for imposing static equilibrium conditions if the model included a full set of futures markets, and if an equilibrium set of prices and quantities across time were computed simultaneously, as is the case in the SRI model. Of course, to point this out is not to make a value judgment about the quality of the SRI model versus the Hnyilicza model, since the emphases of the two models are different.

General equilibrium in the Hnyilicza model

Figure 5 shows the result of collecting together the various supply and demand curves that we have extracted from the production sector, the consumption sector, and the investment sector. The point of intersection of all of these curves coincides with the equilibrium prices and quantities that Hnyilicza's fifty-five simultaneous equations are solved for. We can thus interpret equilibrium in terms of supply-demand balance in the following markets:

1. Capital Services to the Energy Industry
2. Capital Services to the Non-energy Industry
3. Labor Services
4. Energy Imports
5. Non-energy Imports
6. Energy Consumption Goods
7. Non-energy Consumption Goods
8. Energy Intermediate Goods
9. Non-energy Intermediate Goods
10. [Investment Goods]
11. [Savings]

Time-series projections of these equilibrium prices and quantities are the principal outputs of the Hnyilicza model. The following variables are exogenous to the model and are required as inputs:

- Inventory investment
- Labor force
- Tax rates
- Government expenditures
- Exports
- International financial claims
- Government transfer payments (e.g., welfare)
- Rate of unemployment
- Import prices
- Rate of replacement of capital stock
- Initial year private national wealth
- Initial year capital stock
- Quality of capital (a constant, multiplied by capital stock to give capital services)

A note on the new version of the Hudson-Jorgenson model*

Hudson and Jorgenson have recently reworked their original model. Their new model is very similar to the Hnyilicza model, especially as regards the use of the variable coefficient input-output methodology in an economic equilibrium context. Where the two models differ is in the degree of disaggregation of the production sector. As we saw, the Hnyilicza model's production submodel consisted of two sectors: energy and non-energy. The Hudson-Jorgenson production submodel consists of six sectors:

- Coal
- Crude petroleum
- Refined petroleum products
- Electricity
- Natural gas (crude)
- Delivered gas

* The author is grateful to Dr. Edward Hudson for a helpful discussion.
THE SRI NATIONAL ENERGY MODEL

Introduction

The SRI National Energy Model is an economic equilibrium model of the U.S. energy system covering all major fuels from primary resources to end use energy consumption (i.e., energy service demands) over the period 1977 to 2025. The model is regionally disaggregated with nine demand regions (such as New England), twenty primary resource supply regions (such as Alaska North Slope), and 600 transportation and distribution links. Conversion technologies such as coal gasification, crude oil refining, and electric power generation by type of fuel are explicitly modeled. The model computes regional prices and quantities of all energy forms, as well as energy technology requirements over the period of 1977 to 2025 by balancing supply and demand.

The SRI model differs in several important respects from the models considered in the previous section. To begin with, the Hudson-Jorgenson and Hnyilicza models are highly aggregated models of the overall economy. In contrast to this, the SRI model is a very disaggregated and detailed model of the energy sector of the U.S. economy. All this detail can be represented graphically by means of a "spatial network." We shall discuss this network just below. Above and beyond this, the SRI model makes use of a different methodology. While the model is an economic equilibrium model in the sense that it determines prices and quantities which are in supply-demand balance,—just as the other models do—it does not make use of the "translog" methodology. There are no variable input-output coefficients. Rather, economic equilibrium is characterized in a much more direct manner. One solves directly for prices and quantities which are in balance. Moreover, we can visualize "equilibrium" in this model in terms of a supply-demand balance at each point in the spatial network referred to above.

Partly for this reason, we shall start off by discussing the spatial energy network which lies at the heart of the model. Next we shall describe what is meant by "equilibrium" in this context. Finally, we shall gain a deep understanding of the model by analyzing the algorithm used to achieve a supply-demand balance throughout the network.
A spatial network representation of the energy market

The essence of the SRI model is a spatial network representation of the energy system. In the network, the major elements of the U.S. energy system—from the primary resource supplies through the end use energy demands—are described in relationship with each other. A sample energy network is shown in Figure 6. In this figure, the resource supply curves are at the bottom; the usable energy demand curves are at the top. Between these curves is the network describing the entire energy system. The current network has about 2400 materials, processes, and transportation links. A material is a primary resource product, or usable energy form at a specific location. A material is represented by a node in Figure 6. A process represents a sector of the energy industry such as coal mining or gasification at a specific location or a class of consumers using a particular energy-consuming device. A transportation link represents the process of moving a material from one location to another.

To get a sense of the many paths in the network, consider first the path where coal is mined, converted into synthetic (high Btu) gas, piped to a demand center in a demand region, distributed to industrial users, and consumed as boiler fuel to produce steam. The same end use market could be supplied by coal transported by unit train, distributed to the same industrial users, and used in a coal boiler to produce steam. These two paths can be traced in Figure 6. In the SRI National Energy Model, there are 24 end uses (such as industrial steam) in each of the nine demand regions, and thirty primary resource supplies (such as coal) in the various resource basins. The alternative conversion technologies in the model include all important types of electric power generation (in base, intermediate, and peak load applications), crude oil refining, coal gasification, methanol from coal, hydrogen production from coal, transportation, distribution, and end use conversion.

The model explicitly considers the evolution of the energy system through time—it calculates prices and quantities in a number of time periods. It is important to recognize that the solution to this "dynamic" model is not equivalent to using a "static" model in each of the time periods. Rather,
the prices and quantities in each period are described by equations interrelating both past and future prices and quantities.

On the meaning of economic equilibrium in the SRI model

In their earlier research, Brock and Nesbitt have drawn upon economic theory to establish the following results concerning economic equilibrium in spatial networks:

1. There exists an aggregate demand curve and an aggregate supply curve at every material node (circle) in the network;
2. There exists a supply curve and a demand curve on every link in the network. Any such pair characterizes the supply/demand conditions governing the behavior of the (unique) conversion or transportation process attached to the link.

The equilibrium prices and quantities are those for which all aggregate supply/demand curve pairs are simultaneously balanced at all material nodes; or equivalently for which all supply/demand curve pairs are balanced on all the links of the network.

The operation of the model

The SRI model is best described in terms of the algorithm used to solve for equilibrium, and in terms of the relationship of the algorithm to the spatial network and its components. Therefore, in this section we shall describe the algorithm in considerable detail. When we say that this algorithm "solves" the spatial network we simply mean that it determines the set of equilibrium prices and quantities which obtain at each point (link and node) in the network.

The algorithm consists of four essential steps which are repeated at each iteration. The procedure is illustrated at a fairly abstract level in Figure 7. Beginning with an initial "guess" at the equilibrium quantity \( q_{k-1} \), the four steps of the algorithm are:

1. To apply (inverse) resource supply function* \( p_k = S^{-1}(q_{k-1}) \)
2. To proliferate prices up the network (supply function transformation) \( p_{k+1} = T_s[p_k] \)
3. To apply usable energy demand function \( q_{k+1} = D[p_{k+1}] \)
4. To proliferate quantities down the network (demand function transformation) \( q_{k+2} = T_d[q_{k+1}] \)

* An inverse supply function expresses price as a function of quantity.

We will now discuss each of these four steps in some detail, and in a less abstract manner.

1. Inverse Supply Function: In the first step of the algorithm, the inverse supply function describing each of the primary resources at the bottom of the network in Figure 6 is applied. The algorithm begins with a "guess" at the pro-
duction level over time for each of the primary resources (denoted \( q_{k-1}(t) \) as in Figure 7, where the \( t \) denotes a vector over time).

Using the guess \( q_{k-1}(t) \), the inverse supply function is applied to give \( p_k(t) \), the price of the corresponding primary resource. Schematically, Step 1 is given by

\[
p_k(t) = S^{-1}[q_{k-1}(t)]
\]

The inverse supply functions are given exogenously, one for each primary resource.

The first step of the algorithm is to guess initial quantities \( q_a^0, q_b^0 \) of natural gas and synthetic gas respectively and, using the natural gas and synthetic gas supply curves, to calculate the corresponding prices \( p_a^1 \) and \( p_b^1 \) for natural and synthetic gas. These prices apply at the process boxes A and B in Figure 8.

2. Supply Function Transformation: In the second step of the algorithm, the intermediate technology submodels are used to compute product prices given input fuel prices.

The technology models begin by assuming that the measure of profitability to a firm which builds a unit of capacity at time \( t \) is given by the net present value of the cash flow generated by that unit of capacity:

\[
NPV(t) = -c(t) + \sum_{t=1}^{L} \frac{p(T) - \phi(T)}{(1+r)^{T-t}}
\]

where

\[
c(t) = \text{capital cost of the new unit of capacity built at time } t;
\]

\( p(T) = \text{product price at time } T; \)

\( \phi(T) = \text{operating cost (including fuel cost) at time } T; \)

\( r = \text{discount rate (cost of capital) } \)

\( L = \text{life of technology.} \)

In order to simplify our discussion, we will assume that technologies have a useful life of three years (\( L = 3 \)) and that we wish to consider the interrelationship of prices over a three-year time horizon.

It is known from elementary economic theory that at equilibrium, profit maximizing firms earn zero profit. Expressed in terms of net present value, the zero profit assumption is equivalent to:

\[
NPV(t) = 0 \quad t = 1, 2, \ldots
\]

where we have assumed that the cost of debt and equity capital are included in the discount rate \( r \). Assuming \( NPV(t) = 0 \) for new plants built in Years 1, 2, and 3, we can write:

\[
c(1) = p(1) - \phi(1) + \frac{p(2) - \phi(2)}{1+r} + \frac{p(3) - \phi(3)}{(1+r)^2}
\]

\[
c(2) = p(2) - \phi(2) + \frac{p(3) - \phi(3)}{1+r} + \frac{p(4) - \phi(4)}{(1+r)^2}
\]

\[
c(3) = p(3) - \phi(3) + \frac{p(4) - \phi(4)}{1+r} + \frac{p(5) - \phi(5)}{(1+r)^2},
\]

which interrelates the equilibrium prices (and, of course, the capital and operating costs and the discount rate). If the firm which is trying to decide whether to add new capacity at time \( t = 3 \) knew with certainty what future prices \( p(4) \) and \( p(5) \) and future operating costs \( \phi(4) \) and \( \phi(5) \) would be, it could, by rearranging the third equation above, calculate the minimum price in Year 3, \( p(3) \), below which it would not add capacity and above which it would add more capacity.*

\[
p(3) = p(3) + \frac{p(4) - \phi(4)}{1+r} + \frac{p(5) - \phi(5)}{(1+r)^2} = \phi(3) + \text{capital charge}
\]

Knowing \( p(3), \phi(3), p(4), \) and \( \phi(4) \), the second equation of the above three can be used to solve for \( p(2) \). A similar calculation gives \( p(1) \).

To summarize, for each technology, given the fuel prices (contained in \( \phi(t) \)), given assumptions about prices and operating costs past the end of the model horizon, and given process economic data \( c(t) \) and \( \phi(t) \), product prices are calculated recursively backward in time

\[
p(3), p(2), p(1)
\]

as indicated above. When we arrive at the top of the network, we will have calculated the prices \( p_{k+1}(t) \) in Figure 7.

In the second step of the algorithm involves taking the prices \( p_a^0, p_b^0 \) (and the quantities \( q_a^0, q_b^0 \)) and determining the corresponding prices \( p_a^1, p_b^1 \) at the top of the network. The first step is to compute the average price of gas entering the pipeline at \( \alpha \) in Figure 8 as follows:

\[
p = \frac{p_a^0 q_a^0 + p_b^0 q_b^0}{q_a^0 + q_b^0}
\]

The price of gas delivered to the two end users (i.e., to \( \beta \) in the figure) and would simply be

\[
p_\beta \alpha = p_c = \frac{p}{e} + c.
\]

3. Demand Function: The third step in the algorithm is to apply the direct demand function at the top of the network using the prices \( p_{k+1}(t) \) computed on the way up in Step 2. That is, an estimate of demand is obtained by applying the demand function

\[
q_{k+1}(t) = D[p_{k+1}(t)].
\]

Returning to Figure 8, this step of the algorithm involves taking the price \( p_a^0 \) of residential gas determined by Step 2 and finding the residential demand \( q_a^0 \) at that price using the residential demand curve. A similar procedure yields the industrial demand \( q_b^0 \) corresponding to the industrial gas price \( p_b^0 \) determined in Step 2.

4. Demand Function Transformation: In the fourth step of the algorithm, the technology submodels are used to compute fuel requirements given product demands, and market share submodels are used to divide demand among compet-

* Equilibrium would obtain at the price \( p(3) \) itself because the firm would just be indifferent between adding and not adding capacity.
ing technologies based on the price computed for those technologies in Step 3. We will discuss both calculations briefly.

First, each technology is characterized by a thermal efficiency $e(t)$ for new units built at time $t$ and thus by an average efficiency $\bar{e}(t)$ for all plants in place at time $t$. Hence, the fuel required is given by

$$q_{k+2}(t) = q_{k+1}(t)/\bar{e}(t).$$

Second, if the quantity $q_{k+2}(t)$ is to be split among a number of competing suppliers, a "market share" model is applied to the quantity $q_{k+1}(t)$ to split it among competing supply technologies based on the prices computed for those technologies in Step 2. We do not have space enough to discuss this market split model here.

This step of the algorithm is better explained in terms of the network in Figure 8. Referring to that figure, the fourth step of the algorithm actually consists of two operations. First, the residential and industrial quantities $q^1_A$ and $q^1_B$ respectively determined in Step 3 are added to give the total demand for gas $q = q^1_A + q^1_B$ at $\beta$ in the figure. In order for the pipeline to deliver $\bar{q}$ units of gas at $\beta$, it must take in $\bar{q}/e$ units of gas at $\alpha$, where $e$ is the thermal efficiency of the pipeline, including transportation losses.

Given the demand $\bar{q}/e$ of gas at $\alpha$, we then apply the market share model to compute how much of that gas would be synthetic and how much would be natural at the prices $p^*_A$, $p^*_B$ determined in Step 2. That is, we would compute

$$q_A^1 = MS_A(p^*_A, p^*_B) \times \bar{q}/e$$
$$q_B^1 = MS_B(p^*_A, p^*_B) \times \bar{q}/e$$

where $MS_A(p^*_A, p^*_B)$ is the fraction of the demand that would be satisfied by Process $A$ (natural gas) if the prices of natural and synthetic gas were $p^*_A$ and $p^*_B$ respectively, and similarly for Process $B$.

Characterization of equilibrium in the network

If $q_A^1 = q^1_A$ and $q_B^1 = q^1_B$, equilibrium has been achieved (recall that $q^0_A$ and $q^0_B$ were our initial guesses in Step 1). If not, we return to Step 1 with the new guesses $q_A^1$ and $q_B^1$, and repeat the four steps, as many times as necessary to achieve equilibrium.

THE ROLE OF ECONOMIC MODELS IN NORMATIVE POLICY ANALYSIS

We shall conclude the present essay by issuing some precautionary remarks about the role of economic models in normative public policy analyses. The interested reader will find an elaboration of these remarks in Chapter I of Brock and Nesbitt. 5

It must be appreciated that the models discussed in this report are economic in nature. That is, their aim is to project prices and quantities of energy (and, sometimes, non-energy) commodities. Yet normative policy analysis is interested not only in the economic dimension of alternative policies, but also in the political and ethical dimensions.

For example, a policy maker might well wish to have at least probabilistic information about the likely price/quantity impacts of pursuing alternative strategies. For an example of this, see the companion paper presented by Dr. Steven N. Tani. Tani points out how price quantity data generated by an energy model can be used to make economic welfare judgments about alternative policies by means of the criterion of "net social surplus." This is important—indeed, it is far the most important role of strictly economic models in normative policy analyses.

But the same policy maker might also wish to know the political implications of pursuing alternative strategies. For example, a policy which looks good on economic grounds may for various reasons be "politically unacceptable." Game theoretic models can in principle be used to determine which policies are politically acceptable in a balance-of-power sense. But economic equilibrium models are not game theoretic models, and should not be confused as such.

Finally, there are the distributional concerns which are central to normative ethics. Economic planning models in and of themselves cannot furnish us with the answer as to which policy is most "fair." President Carter has repeatedly emphasized his concern with equity. But in this matter we have a problem for ethics—not for price/quantity models. While these points may seem too obvious to need explicit statement there is often confusion about the proper role of economic data in normative policy analysis. For this reason, we feel it worthwhile to close with a reminder. There is one other important limitation to economic models of the kind reviewed above. This concerns the use of an "equilibrium" methodology in the context of regulated commodities such as natural gas. Many problems of interpretation arise here; but they are outside the scope of the present paper.

REFERENCES