INTRODUCTION

The ARPA Network (ARPANET) project brought together many individuals with diverse backgrounds, philosophies, and technical approaches from the fields of computer science, communication theory, operations research and others. The project was aimed at providing an efficient and reliable computer communications system (using message switching techniques) in which computer resources such as programs, data, storage, special purpose hardware etc., could be shared among computers and among many users.38 The variety of design methods, ranging from theoretical modeling to hardware development, were primarily employed independently, although cooperative efforts among designers occurred on occasion. As of November, 1971, the network has been an operational facility for many months, with about 20 participating sites, a network information center accessible via the net, and well over a hundred researchers, system programmers, computer center directors and other technical and administrative personnel involved in its operation.

In this paper, we review and evaluate the methods used in the ARPANET design from the vantage of over two years' experience in the development of the network. In writing this paper, the authors have each made equal contributions during a series of intensive discussions and debates. Rather than present merely a summary of the procedures that were used in the network design, we have attempted to evaluate each other's methods to determine their advantages and drawbacks. Our approaches and philosophies have often differed radically and, as a result, this has not been an easy or undisturbing process. On the other hand, we have found our collaboration to be extremely rewarding and, notably, we have arrived at many similar conclusions about the network's behavior that seem to be generally applicable to message switched networks.

The essence of a network is its design philosophy, its performance characteristics, and its cost of implementation and operation. Unfortunately, there is no generally accepted definition of an “optimal” network or even of a “good” network. For example, a network designed to transmit large amounts of data only during late evening hours might call for structural and performance characteristics far different from one servicing large numbers of users who are rapidly exchanging short messages during business hours. We expect this topic, and others such as the merits of message switching vs. circuit switching or distributed vs. centralized control to be a subject of discussion for many years.1,14,24,32,37

A cost analysis performed in 1967-1968 for the ARPA Network indicated that the use of message switching would lead to more economical communications and better overall availability and utilization of resources than other methods.36,38 In addition to its impact on the availability of computer resources, this decision has generated widespread interest in store-and-forward communications. In many instances, the use of store-and-forward communication techniques can result in
greater flexibility, higher reliability, significant technical advantage, and substantial economic savings over the use of conventional common carrier offerings. An obvious trend toward increased computer and communication interaction has begun. In addition to the ARPANET, research in several laboratories is under way, small experimental networks are being built, and a few examples of other government and commercial networks are already apparent.\(^{{6,7,31,40,47,48,52}}\)

In the ARPANET, each time-sharing or batch processing computer, called a Host, is connected to a small computer called an Interface Message Processor (IMP). The IMPs, which are interconnected by leased 50 kilobit/second circuits, handle all network communication for their Hosts. To send a message to another Host, a Host precedes the text of its message with an address and simply delivers it to its IMP. The IMPs then determine the route, provide error control, and notify the sender of its receipt. The collection of Hosts, IMPs, and circuits forms the message switched resource sharing network. A good description of the ARPANET, and some early results on protocol development and modeling are given in References 3, 12, 15, 23 and 38. Some experimental utilization of the ARPANET is described in Reference 42. A more recent evaluation of such networks and a forward look is given in References 35 and 39.

The development of the Network involved four principal activities:

1. The design of the IMPs to act as nodal store-and-forward switches,
2. The topological design to specify the capacity and location of each communication circuit within the network,
3. The design of higher level protocols for the use of the network by time-sharing, batch processing and other data processing systems, and

Each of the first three activities were essentially performed independently of each other, whereas the modeling effort partly affected the IMP design effort, and closely interacted with the topological design project.

The IMPs were designed by Bolt Beranek and Newman Inc. (BBN) and were built to operate independent of the exact network connectivity; the topological structure was specified by Network Analysis Corporation (NAC) using models of network performance developed by NAC and by the University of California at Los Angeles (UCLA). The major efforts in the area of system modeling were performed at UCLA using theoretical and simulation techniques. Network performance measurements have been conducted during the development of the network by BBN and by the Network Measurement Center at UCLA. To facilitate effective use of the net, higher level (user) protocols are under development by a group of representatives of universities and research centers. This group, known as the Network Working Group, has already specified a Host to Host protocol and a Telnet protocol, and is in the process of completing other function oriented protocols.\(^{{4,29}}\) We make no attempt to elaborate on the Host to Host protocol design problems in this paper.

THE NETWORK DESIGN PROBLEM

A variety of performance requirements and system constraints were considered in the design of the net. Unfortunately, many of the key design objectives had to be specified long before the actual user requirements could be known. Once the decision to employ message switching was made, and fifty kilobit/second circuits were chosen, the critical design variables were the network operating procedure and the network topology; the desired values of throughput, delay, reliability and cost were system performance and constraint variables. Other constraints affected the structure of the network, but not its overall properties, such as those arising from decisions about the length of time a message could remain within the network, the location of IMPs relative to location of Hosts, and the number of Hosts to be handled by a single IMP.

In this section, we identify the central issues related to IMP design, topological design, and network modeling. In the remainder of the paper, we describe the major design techniques which have evolved.

**IMP properties**

The key issue in the design of the IMPs was the definition of a relationship between the IMP subnet and the Hosts to partition responsibilities so that reliable and efficient operation would be achieved. The decision was made to build an autonomous subnet, independent (as much as possible) of the operation of any Host. The subnet was designed to function as a "communications system"; issues concerning the use of the subnet by the Hosts (such as protocol development) were initially left to the Hosts. For reliability, the IMPs were designed to be robust against all line failures and the vast majority of IMP and Host failures. This decision required routing strategies that dynamically adapt to changes in the states of IMPs and circuits,
and an elaborate flow control strategy to protect the subnet against Host malfunction and congestion due to IMP buffer limitations. In addition, a statistics and status reporting mechanism was needed to monitor the behavior of the network.

The number of circuits that an IMP must handle is a design constraint directly affecting both the structure of the IMP and the topological design. The speed of the IMP and the required storage for program and buffers depend directly upon the total required processing capacity, which must be high enough to switch traffic from one line to another when all are fully occupied. Of great importance is the property that all IMPs operate with identical programs. This technique greatly simplifies the problem of network planning and maintenance and makes network modifications easy to perform.

The detailed physical structure of the IMP is not discussed in this paper. However, the operating procedure, which guides packets through the net, is very much of interest here. The flow control, routing, and error control techniques are integral parts of the operating procedure and can be studied apart from the hardware by which they are implemented. Most hardware modifications require changes to many IMPs already installed in the field, while a change in the operating procedure can often be made more conveniently by a change to the single operating program common to all IMPs, which can then be propagated from a single location via the net.

Topological properties

The topological design resulted in the specification of the location and capacity of all circuits in the network. Projected Host—Host traffic estimates were known at the start to be either unreliable or wrong. Therefore, the network was designed under the assumption of equal traffic between all pairs of nodes. (Additional superimposed traffic was sometimes included for those nodes with expectation of higher traffic requirements.)

The topological structure was determined with the aid of specially developed heuristic programs to achieve a low cost, reliable network with a high throughput and a general insensitivity to the exact traffic distribution. Currently, only 50 kilobit/second circuits are being used in the ARPANET. This speed line was chosen to allow rapid transmission of short messages for interactive processing (e.g., less than 0.2 seconds average packet delay) as well as to achieve high throughput (e.g., at least 50 kilobits/second) for transmission of long messages. For reliability, the network was constrained to have at least two independent paths between each pair of IMPs.

The topological design problem requires consideration of the following two questions:

(1) Starting with a given state of the network topology, what circuit modifications are required to add or delete a set of IMPs?

(2) Starting with a given state of network topology, when and where should circuits be added or deleted to account for long term changes in network traffic?

If the locations of all network nodes are known in advance, it is clearly most efficient to design the topological structure as a single global effort. However, in the ARPANET, as in most actual networks, the initial designation of node locations is modified on numerous occasions. On each such occasion, the topology can be completely reoptimized to determine a new set of circuit locations.

In practice, there is a long lead time between the ordering and the delivery of a circuit, and major topological modifications cannot be made without substantial difficulty. It is therefore prudent to add or delete nodes with as little disturbance as possible to the basic network structure consistent with overall economical operation. Figure 1 shows the evolution of the ARPANET from the basic four IMP design in 1969 to the presently planned 27 IMP version. Inspection of the 24 and 27 IMP network designs reveals a few substantial changes in topology that take advantage of the new nodes being added. Surprisingly enough, a complete “reoptimization” of the 27 IMP topology yields a network only slightly less expensive (about 1 percent) than the present network design.

Network models

The development of an accurate mathematical model for the evaluation of time delay in computer networks is among the more difficult of the topics discussed in this paper. On the one hand, the model must properly reflect the relevant features of the network structure and operation, including practical constraints. On the other hand, the model must result in a mathematical formulation which is tractable and from which meaningful results can be extracted. However, the two requirements are often incompatible and we search for an acceptable compromise between these two extremes.

The major modeling effort thus far has been the study of the behavior of networks of queues. This emphasis is logical since in message switched systems, messages experience queueing delays as they pass from node to node and thus a significant performance measure is the
speed at which messages can be delivered. The queueing models were developed at a time when there were no operational networks available for experimentation and model validation, and simulation was the only tool capable of testing their validity. The models, which at all times were recognized to be idealized statements about the real network, were nonetheless crucial to the ARPANET topological design effort since they afforded the only known way to quantitatively predict the properties of different routing schemes and topological structures. The models have been subsequently demonstrated to be very accurate predictors of network throughput and indispensable in providing analytical insight into the network’s behavior.

The key to the successful development of tractable models has been to factor the problem into a set of simpler queueing problems. There are also heuristic design procedures that one can use in this case. These procedures seem to work quite well and are described later in the paper. However, if one specializes the problem and removes some of the real constraints, theory and analysis become useful to provide understanding, intuition and design guidelines for the original constrained problem. This approach uncovers global properties of network behavior, which provide keys to good heuristic design procedures and ideal performance bounds.

DESIGN TECHNIQUES

In this section we describe the approaches taken to the design problems introduced in the previous section. We first summarize the important properties of the ARPANET design:

(1) A communications cost of less than 30 cents per thousand packets (approximately a megabit).
(2) Average packet delays under 0.2 seconds through the net.
(3) Capacity for expansion to 64 IMPs without major hardware or software redesign.
(4) Average total throughput capability of 10-15 kilobits/second for all Hosts at an IMP.
(5) Peak throughput capability of 85 kilobits/second per pair of IMPs in an otherwise unloaded network.
(6) Transparent communications with maximum message size of approximately 8000 bits and error rates of one bit in $10^{12}$ or less.
(7) Approximately 98 percent availability of any IMP and close to 100 percent availability of all operating IMPs from any operable IMP.

The relationships between the various design efforts are illustrated by these properties. The topological design provides for both a desired average throughput and for two or more paths to be fully used for communication between any pair of Hosts. The operating procedure should allow any pair of Hosts to achieve those objectives. The availability of IMPs to communicate reflects both the fact that IMPs are down about 2 percent of the time and that the topology is selected so that circuit failures contribute little additional to the total system downtime.

**IMP design**

The IMP design consists of two closely coupled but nonetheless separable pieces—the physical hardware specification (based on timing and reliability considerations and the operating procedure) and the design and implementation of the operating procedure using the specified IMP hardware. The IMP originally developed for the ARPANET contains a 16-bit one microsecond computer that can handle a total of about \( \frac{3}{4} \) megabits/second of “useful” information on a total of approximately one megabit/second of circuit capacity (e.g., twenty 50 kilobit/second circuits). Hardware is likely to change as a function of the required IMP capacity but an operating procedure that operates well at one IMP capacity is likely to be transferable to machines that provide different capacity. However, as a network grows in size and utilization, a more comprehensive operating procedure that takes account of known structural properties, such as a hierarchical topology, is appropriate.

Four primary areas of IMP design, namely message handling and buffering, error control, flow control, and routing are discussed in this section. The IMP provides buffering to handle messages for its Host and packets for other IMPs. Error control is required to provide reliable communication of Host messages in the presence of noisy communication circuits. The design of the operating procedure should allow high throughput in the net under heavy traffic loads. Two potential obstacles to achieving this objective are: (1) The net can become congested and cause the throughput to decrease with increasing load, and (2) The routing procedure may be unable to always adapt sufficiently fast to the rapid movement of packets to insure efficient routing. A flow control and routing procedure is needed that can efficiently meet this requirement.

**Message handling and buffering**

In the ARPANET, the maximum message size was constrained to be approximately 8000 bits. A pair of Hosts will typically communicate over the net via a sequence of transmitted messages. To obtain delays of a few tenths of a second for such messages and to lower the required IMP buffer storage, the IMP program partitions each message into one or more packets each containing at most approximately 1000 bits. Each packet of a message is transmitted independently to the destination where the message is reassembled by the IMP before shipment to that destination Host. Alternately, the Hosts could assume the responsibility for reassembling messages. For an asynchronous IMP-Host channel, this marginally simplifies the IMP’s task. However, if every IMP-Host channel were synchronous, and the Host provided the reassembly, the IMP task can be further simplified. In this latter case, “IMP-like” software would have to be provided in each Host.

The method of handling buffers should be simple to allow for fast processing and a small amount of program. The number of buffers should be sufficient to store enough packets for the circuits to be used to capacity; the size of the buffers may be intuitively selected with the aid of simple analytical techniques. For example, fixed buffer sizes are useful in the IMP for simplicity of design and speed of operation, but inefficient utilization can arise because of variable length packets. If each buffer contains \( A \) words of overhead and provides space for \( M \) words of text, and if message sizes are uniformly distributed between 1 and \( L \), it can be shown that the choice of \( M \) that minimizes the expected storage is approximately \( \sqrt{AL} \). In practice, \( M \) is chosen to be somewhat smaller on the assumption that most traffic will be short and that the amount of overhead can be as much as, say, 25 percent of buffer storage.

**Error control**

The IMPs must assume the responsibility for providing error control. There are four possibilities to consider:

1. Messages are delivered to their destination out of order.
2. Duplicate messages are delivered to the destination.
3. Messages are delivered with errors.
4. Messages are not delivered.
The task of proper sequencing of messages for delivery to the destination Host actually falls in the province of both error control and flow control. If at most one message at a time is allowed in the net between a pair of Hosts, proper sequencing occurs naturally. A duplicate packet will arrive at the destination IMP after an acknowledgment has been missed, thus causing a successfully received packet to be retransmitted. The IMPs can handle the first two conditions by assigning a sequence number to each packet as it enters the network and processing the sequence number at the destination IMP. A Host that performs reassembly can also assign and process sequence numbers and check for duplicate packets. For many applications, the order of delivery to the destination is immaterial. For priority messages, however, it is typically the case that fast delivery requires a perturbation to the sequence.

Errors are primarily caused by noise on the communication circuits and are handled most simply by error detection and retransmission between each pair of IMPs along the transmission path. This technique requires extra storage in the IMP if either circuit speeds or circuit lengths substantially increase. Failures in detecting errors can be made to occur on the order of years to centuries apart with little extra overhead (20-30 parity bits per packet with the 50 kilobit/second circuits in the ARPANET). Standard cyclic error detection codes have been usefully applied here.

A reliable system design insures that each transmitted message is accurately delivered to its intended destination. The occasional time when an IMP fails and destroys a useful in-transit message is likely to occur far less often than a similar failure in the Hosts and has proven to be unimportant in practice, as are errors due to IMP memory failures. A simple end to end retransmission strategy will protect against these situations, if the practical need should arise. However, the IMPs are designed so that they can be removed from the network without destroying their internally stored packets.

**Flow control**

A network in which packets may freely enter and leave can become congested or logically deadlocked and cause the movement of traffic to halt. Flow control techniques are required to prevent these conditions from occurring. The provision of extra buffer storage will mitigate against congestion and deadlocks, but cannot in general prevent them.

The sustained failure of a destination Host to accept packets from its IMP at the rate of arrival will cause the net to fill up and become congested. Two kinds of logical deadlocks, known as reassembly lockup and store-and-forward lockup may also occur. In reassembly lockup, the remaining packets of partially reassembled messages are blocked from reaching the destination IMP (thus preventing the message from being completed and the reassembly space freed) by other packets in the net that are waiting for reassembly space at that destination to become free. In a store-and-forward lockup, the destination has room to accept arriving packets, but the packets interfere with each other by tying up buffers in transit in such a way that none of the packets are able to reach the destination. These phenomena have only been made to occur during very carefully arranged testing of the ARPANET and by simulation.

In the original ARPANET design, the use of software links and RFNMS protected against congestion by a single link or a small set of links. However, the combined traffic on a large number of links could still produce congestion. Although this strategy did not protect against lockup, the method has provided ample protection for the levels of traffic encountered by the net to date.

A particularly simple flow control algorithm that augments the original IMP design to prevent congestion and lockup is also described in Reference 17. This scheme includes a mechanism whereby packets may be discarded from the net at the destination IMP when congestion is about to occur, with a copy of each discarded packet to be retransmitted a short time later by the originating Host's IMP. Rather than experience excessive delays within the net as traffic levels are increased, the traffic is queued outside the net so that the transit time delays internal to the net continue to remain small. This strategy prevents the insertion of more traffic into the net than it can handle.

It is important to note the dual requirement for small delays for interactive traffic and high bandwidth for the fast transfer of files. To allow high bandwidth between a pair of Hosts, the net must be able to accept a steady flow of packets from one Host and at the same time be able to rapidly quench the flow at the entrance to the source IMP in the event of imminent congestion at the destination. This usually requires that a separate provision be made in the algorithm to protect short interactive messages from experiencing unnecessarily high delays.

**Routing**

Network routing strategies for distributed networks require routing decisions to be made with only information available to an IMP and the IMP must
execute those decisions to effect the routing. A simple example of such a strategy is to have each IMP handling a packet independently route it along its current estimate of the shortest path to the destination.

For many applications, it suffices to deal with an idealized routing strategy which may not simulate the IMP routing functions in detail or which uses information not available to the IMP. The general properties of both strategies are usually similar, differing mainly in certain implementation details such as the availability of buffers or the constraint of counters and the need for the routing to quickly adapt to changes in IMP and circuit status.

The IMPs perform the routing computations using information received from other IMPs and local information such as the alive/dead state of its circuits. In the normal case of time varying loads, local information alone, such as the length of internal queues, is insufficient to provide an efficient routing strategy without assistance from the neighboring IMPs. It is possible to obtain sufficient information from the neighbors to provide efficient routing, with a small amount of computation needed per IMP and without each IMP requiring a topological map of the network. In certain applications where traffic patterns exhibit regularity, the use of a central controller might be preferable. However, for most applications which involve dynamically varying traffic flow, it appears that a central controller cannot be used more effectively than the IMPs to update routing tables if such a controller is constrained to derive its information via the network. It is also a less reliable approach to routing than to distribute the routing decisions among the IMPs.

The routing information cannot be propagated about the net in sufficient time to accurately characterize the instantaneous traffic flow. An efficient algorithm, therefore, should not focus on the movement of individual packets, but rather use topological or statistical information in the selection of routes. For example, by using an averaging procedure, the flow of traffic can be made to build up smoothly. This allows the routing algorithm ample time to adjust its tables in each IMP in advance of the build-up of traffic.

The scheme originally used in the ARPA network had each IMP select one output line per destination onto which to route packets. The line was chosen to be the one with minimum estimated time delay to the destination. The selection was updated every half second using minimum time estimates from the neighboring IMPs and internal estimates of the delay to each of the neighbors. Even though the routing algorithm only selects one line at a time per destination, two output lines will be used if a queue of packets waiting transmission on one line builds up before the routing update occurs and another line is chosen. Modifications to the scheme which allow several lines per destination to be used in an update interval (during which the routing is not changed) are possible using two or more time delay estimates to select the paths.

In practice, this approach has worked quite effectively with the moderate levels of traffic experienced in the net. For heavy traffic flow, this strategy may be inefficient, since the routing information is based on the length of queues, which we have seen can change much faster than the information about the change can be distributed. Fortunately, this information is still usable, although it can be substantially out of date and will not, in general, be helpful in making efficient routing decisions in the heavy traffic case.

A more intricate scheme, recently developed by BBN, allows multiple paths to be efficiently used even during heavy traffic. Preliminary simulation studies indicate that it can be tailored to provide efficient routing in a network with a variety of heavy traffic conditions. This method separates the problem of defining routes onto which packets may be routed from the problem of selecting a route when a particular packet must be routed. By this technique, it is possible to send packets down a path with the fewest IMPs and excess capacity, or when that path is filled, the one with the next fewest IMPs and excess capacity, etc.

A similar approach to routing was independently derived by NAC using an idealized method that did not require the IMPs to participate in the routing decisions. Another approach using a flow deviation technique has recently been under study at UCLA. The intricacies of the exact approach lead to a metering procedure that allows the overall network flow to be changed slowly for stability and to perturb existing flow patterns to obtain an increased flow. These approaches all possess, in common, essential ingredients of a desirable routing strategy.

**Topological considerations**

An efficient topological design provides a high throughput for a given cost. Although many measures of throughput are possible, a convenient one is the average amount of traffic that a single IMP can send into the network when all other IMPs are transmitting according to a specified traffic pattern. Often, it is assumed that all other IMPs are behaving identically and each IMP is sending equal amounts of traffic to each other IMP. The constraints on the topological design are the available common carrier circuits, the target cost or throughput, the desired reliability, and...
the cost of computation required to perform the topological design.

Since, there was no clear specification of the amount of traffic that the network would have to accommodate initially, it was first constructed with enough excess capacity to accommodate any reasonable traffic requirements. Then as new IMPs were added to the system, the capacity was and is still being systematically reduced until the traffic level occupies a substantial fraction of the network's total capacity. At this point, the net's capacity will be increased to maintain the desired percentage of loading. At the initial stages of network design, the "two-connected" reliability constraint essentially determined a minimum value of maximum throughput. This constraint forces the average throughput to be in the range 10-15 kilobits per second per IMP, when 50 kilobit/second circuits are used throughout the network, since two communication paths between every pair of IMPs are needed. Alternatively, if this level of throughput is required, then the reliability specification of "two-connectivity" can be obtained without additional cost.

Reliability computations

A simple and natural characterization of network reliability is the ability of the network to sustain communication between all operable pairs of IMPs. For design purposes, the requirement of two independent paths between nodes insures that at least two IMPs and/or circuits must fail before any pair of operable IMPs cannot communicate. This criterion is independent of the properties of the IMPs and circuits, does not take into account the "degree" of disruption that may occur and hence, does not reflect the actual availability of resources in the network. A more meaningful measure is the average fraction of IMP pairs that cannot communicate because of IMP and circuit failures. This calculation requires knowledge of the IMP and circuit failure rates, and could not be performed until enough operating data was gathered to make valid predictions.

To calculate network reliability, we must consider elementary network structures known as cutsets. A
cutset is a set of circuits and/or IMPs whose removal from the network breaks all communication paths between at least two operable IMPs. To calculate reliability, it is often the case that all cutsets must be either enumerated or estimated. As an example, in a 23 IMP, 28 circuit ARPA Network design similar to the one shown in Figure 1(d), there are over twenty million ways of deleting only circuits so that the remaining network has at least one operable pair of IMPs with no intact communication paths. Table 1 indicates the numbers of cutsets in the 23 IMP network as a function of the number of circuits they contain.

A combination of analysis and simulation can be used to compute the average fraction of non-communicating IMP pairs. Detailed descriptions of the analysis methods are given in Reference 44 while their application to the analysis of the ARPANET is discussed in Reference 43. The results of an analysis of the 23 IMP version of the network are shown in Figure 2. The curve marked A shows the results under the assumption that IMPs do not fail, while the curve marked B shows the case where circuits do not fail. The curve marked C assumes that both IMPs and circuits fail with equal probability. In actual operation, the average failure probability of both IMPs and circuits is about 0.02. For this value, it can be seen that the effect of circuit failures is far less significant than the effect of IMP failures. If an IMP fails in a network with $n$ IMPs, at least $n-1$ other IMPs cannot communicate with it. Thus, good network design cannot improve upon the effect directly due to IMP failures, which in the ARPANET is the major factor affecting the reliability of the communications. Further, more intricate reliability analyses which consider the loss of throughput capacity because of circuit failures have also been performed and these losses have been shown to be negligible. Finally, unequal failure rates due to differences in line lengths have been shown to have only minor effects on the analysis and can usually be neglected.

**Topological optimization**

During the computer optimization process, the reliability of the topology is assumed to be acceptable if the network is at least two-connected. The object of the optimization is to decrease the ratio of cost to throughput subject to an overall cost limitation. This technique employs a sophisticated network optimization program that utilizes circuit exchange heuristics, routing and flow analysis algorithms, to generate low cost designs. In addition, two time delay models were initially used to (1) calculate the throughput corresponding to an average time delay of 0.2 seconds, (2) estimate the packet rejection rate due to all buffers filling at an IMP. As experience with these models grew, the packet rejection rate was found to be negligible and the computation discontinued. The delay computation (Equation (7) in a later section) was subsequently first replaced by a heuristic calculation to speed the computation and later eliminated after it was found that time delays could be guaranteed to be acceptably low by preventing cutsets from being saturated. This “threshold” behavior is discussed further in the next section.

The basic method of optimization was described in Reference 12 while extensions to the design of large networks are discussed in Reference 9. The method operates by initially generating, either manually or by computer, a “starting network” that satisfies the overall network constraints but is not, in general, a low cost network. The computer then iteratively modifies the starting network in simple steps until a lower cost network is found that satisfies the constraints or the process is terminated. The process is repeated until no further improvements can be found. Using a different starting network can result in a different solution. However, by incorporating sensible heuristics and by using a variety of carefully chosen starting networks and some degree of man-machine interaction, “excellent” final networks usually result. Experience has shown that there are a wide variety of such networks with different topological structures but similar cost and performance.

The key to this design effort is the heuristic procedure by which the iterative network modifications are made. The method used in the ARPANET design involves the removal and addition of one or two circuits at a time. Many methods have been employed, at various times, to identify the appropriate circuits for potential addition or deletion. For example, to delete uneconomical circuits a straightforward procedure simply deletes single circuits in numerical order, reroutes traffic and reevaluates cost until a decrease in cost per megabit is found. At this point, the deletion is made permanent and the process begins again. A somewhat more sophisticated procedure deletes circuits in order of increasing utilization, while a more complex method attempts to evaluate the effect of the removal of any circuit before any deletion is attempted. The circuit with the greatest likelihood of an improvement is then considered for removal and so on.

There are a huge number of reasonable heuristics for circuit exchanges. After a great deal of experimentation with many of these, it appears that the choice of a particular heuristic is not critical. Instead, the speed and efficiency with which potential exchanges can be
investigated appears to be the limiting factor affecting the quality of the final design. Finally, as the size of the network increases, the greater the cost becomes to perform any circuit exchange optimization. Decomposition of the network design into regions becomes necessary and additional heuristics are needed to determine effective decompositions. It presently appears that these methods can be used to design relatively efficient networks with a few hundred IMPs while substantially new procedures will be necessary for networks of greater size.

The topological design requires a routing algorithm to evaluate the throughput capability of any given network. Its properties must reflect those of an implementable routing algorithm, for example, within the ARPANET. Although the routing problem can be formulated as a "multicommodity flow problem" and solved by linear programming for networks with 20-30 IMPs, faster techniques are needed when the routing algorithm is incorporated in a design procedure. The design procedure for the ARPA Network topology iteratively analyzes thousands of networks. To satisfy the requirements for speed, an algorithm which selects the least utilized path with the minimum number of IMPs was initially used. This algorithm was later replaced by one which sends as much traffic as possible along such paths until one or more circuits approach a few percent of full utilization. These highly utilized circuits are then no longer allowed to carry additional flow. Instead, new paths with excess capacity and possibly more intermediate nodes are found. The procedure continues until some cutset contains only nearly fully utilized circuits. At this point no additional flow can be sent. For design purposes, this algorithm is a highly satisfactory replacement for the more complicated multi-commodity approach. Using the algorithm, it has been shown that the throughput capabilities of the ARPA Network are substantially insensitive to the distribution of traffic and depend mainly only on the total traffic flow within the network.

Analytic models of network performance

The effort to determine analytic models of system performance has proceeded in two phases: (1) the prediction of average time delay encountered by a message as it passes through the network, and (2) the use of these queueing models to calculate optimum channel capacity assignments for minimum possible delay. The model used as a standard for the average message delay was first described in Reference 21 where it served to predict delays in stochastic communication networks. In Reference 22, it was modified to describe the behavior of ARPA-like computer networks while in Reference 23 it was refined further to apply directly to the ARPANET.

The single server model

Queueing theory provides an effective set of analytical tools for studying packet delay. Much of this theory considers systems in which messages place demands for transmission (service) upon a single communication channel (the single server). These systems are characterized by $A(\tau)$, the distribution of interarrival times between demands and $B(t)$, the distribution of service times. When the average demand for service is less than the capacity of the channel, the system is said to be stable.

When $A(\tau)$ is exponential (i.e., Poisson arrivals), and messages are transmitted on a first-come first-served basis, the average time $T$ in the stable system is

$$T = \frac{\lambda \bar{t}}{2(1-\rho)} + \bar{t}$$

where $\lambda$ is the average arrival rate of messages, $\bar{t}$ and $\bar{t}^2$ are the first and second moments of $B(t)$ respectively, and $\rho = \lambda < 1$. If the service time is also exponential,

$$T = \frac{\bar{t}}{1-\rho}$$

When $A(\tau)$ and $B(t)$ are arbitrary distributions, the situation becomes complex and only weak results are available. For example, no expression is available for $T$; however the following upper bound yields an excellent approximation as $\rho \to 1$:

$$T \leq \frac{\lambda(\sigma_x^2 + \sigma_y^2)}{2(1-\rho)} + \bar{t}$$

where $\sigma_x^2$ and $\sigma_y^2$ are the variance of the interarrival time and service time distributions, respectively.

Networks of queues

Multiple channels in a network environment give rise to queueing problems that are far more difficult to solve than single server systems. For example, the variability in the choice of source and destination for a message is a network phenomenon which contributes to delay. A principal analytical difficulty results from the fact that flows throughout the network are correlated. The basic approach to solving these stochastic network
problems is to decompose them into analyzable single-server problems which reflect the original network structure and traffic flow.

Early studies of queueing networks indicated that such a decomposition was possible; however, those results do not carry over to message switched computer networks due to the correlation of traffic flows. In Reference 21 it was shown for a wide variety of communication nets that this correlation could be removed by considering the length of a given packet to be an independent random variable as it passes from node to node. Although this "independence" assumption is not physically realistic, it results in a mathematically tractable model which does not seem to affect the accuracy of the predicted time delays. As the size and connectivity of the network increases, the assumption becomes increasingly more realistic. With this assumption, a successful decomposition which permits a channel-by-channel analysis is possible, as follows.

The packet delay is defined as the time which a packet spends in the network from its entry until it reaches its destination. The average packet delay is denoted as $T$. Let $Z_{jk}$ be the average delay for those packets whose origin is IMP $j$ and whose destination is IMP $k$. We assume a Poisson arrival process for such packets with an average of $\gamma_{jk}$ packets per second and an exponential distribution of packet lengths with an average of $1/\mu$ bits per packet. With these definitions, if $\gamma$ is the sum of the quantities $\gamma_{jk}$, then

$$T = \sum_{jk} \frac{\gamma_{jk}}{\gamma} Z_{jk} \tag{4}$$

Let us now reformulate Equation (4) in terms of single channel delays. We first define the following quantities for the $i$th channel: $C_i$ as its capacity (bits/second); $\lambda_i$ as the average packet traffic it carries (packets/second); and $T_i$ as the average time a packet spends waiting for and using the $i$th channel. By relating the $\lambda_i$ to the $\gamma_{jk}$ via the paths selected by the routing algorithm, it is easy to see that

$$T = \sum \frac{\lambda_i}{\gamma} T_i \tag{5}$$

With the assumption of Poisson traffic and exponential service times, the quantities $T_i$ are given by Equation (2). For an average packet length of $1/\mu$ bits, $C_i$ seconds and thus

$$T_i = \frac{1}{\mu C_i - \lambda_i} \tag{6}$$

Thus we have successfully decomposed the analysis problem into a set of simple single-channel problems.

A refinement of the decomposition permits a non-exponential packet length distribution and uses Equation (1) rather than Equation (2) to calculate $T_i$; as an approximation, the Markovian character of the traffic is assumed to be preserved. Furthermore, for computer networks we include the effect of propagation time and overhead traffic to obtain the following equation for average packet delay

$$T = K + \sum \frac{\lambda_i}{\mu C_i} + \frac{\lambda_i/\mu C_i}{\mu C_i - \lambda_i} + P_i + K \tag{7}$$

Here, $1/\mu$ represents the average length of a Host packet, and $1/\mu$ represents the average length of all packets (including acknowledgments, headers, requests for next messages, parity checks, etc.) within the network. The expression $1/\mu C_i + [(\lambda_i/\mu C_i)/(\mu C_i - \lambda_i)] + P_i$ represents the average packet delay on the $i$th channel. The term $(\lambda_i/\mu C_i)/(\mu C_i - \lambda_i)$ is the average time a packet spends waiting at the IMP for the $i$th channel to become available. Since the packet must compete with acknowledgments and other overhead traffic, the overall average packet length $1/\mu$ appears in the expression. The term $1/\mu C_i$ is the time required to transmit a packet of average length $1/\mu$. Finally: $K$ is the nodal processing time, assumed constant, and for the ARPA IMP approximately equal to 0.35 ms; $P_i$ is the propagation time on the $i$th channel (about 20 ms for a 3000 mile channel).

Assuming a relatively homogeneous set of $C_i$ and $P_i$, no individual term in the expression for delay will dominate the summation until the flow $\lambda_i/\mu$ in one channel (say channel $i_j$) approaches the capacity $C_{i_j}$. At that point, the term $T_{i_j}$ and hence $T$ will grow rapidly. The expression for delay is then dominated by one (or more) terms and exhibits a threshold behavior. Prior to this threshold, $T$ remains relatively constant.

The accuracy of the time delay model, as well as this threshold phenomenon was demonstrated on a 19 node network and on the ten node ARPA net derived from Figure 1 (c) by deleting the rightmost five IMPs. Using the routing procedure described in the last section and equal traffic between all node pairs, the channel flows $\lambda_i$ were found for the ten node net and the delay curves shown in Figure 3 were obtained. Curve $A$ was obtained with fixed 1000 bit packets, while curve $B$ was generated for exponentially distributed variable length packets with average size of 500 bits. In both cases $A$ and $B$, all overhead factors were ignored. Note that the delay remains small until a

* In case A, the application of Equation (1) allows for constant packet lengths (i.e., zero variance).
total throughput slightly greater than 400 kilobits/second is reached. The delay then increases rapidly. Curves $C$ and $D$ respectively represent the same situations when the overhead of 136 bits per packet and per RFNM and 152 bits per acknowledgment are included. Notice that the total throughput per IMP is reduced to 250 kilobits/second in case $C$ and to approximately 200 kilobits/second in case $D$.

In the same figure, we have illustrated with x's the results of a simulation performed with a realistic routing and metering strategy. The simulation omitted all network overhead and assumed fixed lengths of 1000 bits for all packets.

It is difficult to develop a practical routing and flow control procedure that will allow each IMP to input identical amounts of traffic. To compare the delay curve $A$ with the points obtained by simulation, the curve should actually be recomputed for the slightly skewed distribution that resulted. It is notable that the delay estimates from the simulation (which used a dynamic routing strategy) and the computation (which used a static routing strategy and the time delay formula) are in close agreement. In particular, they both accurately determined the vertical rise of the delay curve in the range just above 400 kilobits/second, the formula by predicting infinite delay and the simulation by rejecting the further input of traffic.

In practice and from the analytic and simulation studies of the ARPANET, the average queuing delay is observed to remain small (almost that of an unloaded net) and well within the design constraint of 0.2 seconds until the traffic within the network approaches the capacity of a cutset. The delay then increases rapidly. Thus, as long as traffic is low enough and the routing adaptive enough to avoid the premature saturation of cutsets by guiding traffic along paths with excess capacity, queuing delays are not significant.

### Channel capacity optimization

One of the most difficult design problems is the optimal selection of capacities from a finite set of options. Although there are many heuristic approaches to this problem, analytic results are relatively scarce. (For the specialized case of centralized networks, an algorithm yielding optimal results is available.) While it is possible to find an economical assignment of discrete capacities for, say, a 200 IMP network, very little is known about the relation between such capacity assignments, message delay, and cost.

To obtain theoretical properties of optimal capacity assignments, one may ignore the constraint that capacities are obtainable only in discrete sizes. In Reference 21 such a problem was posed where the network topology and average traffic flow were assumed to be known and fixed and an optimal match of capacities to traffic flow was found. Also, the traffic was assumed to be Markovian (Poisson arrivals and exponential packet lengths) and the independence assumption and decomposition method were applied. For each channel, the capacity $C_i$ was found which minimized the average message delay $T_i$ at a fixed total system cost $D$. Since $\lambda_i/\mu$ is the average bit rate on the $i$th channel, the solution to any optimal assignment problem must provide more than this minimal capacity to each channel. This is clear since both Equations (6) and (7) indicate that $T_i$ will become arbitrarily large with less than (or equal to) this amount of capacity. It is not critical exactly how the excess capacity is
assigned, as long as \( C_i > \lambda_i / \mu \). Other important parameters and insights have been identified in studying the continuous capacity optimization problem. For example, the number of excess dollars, \( D_n \), remaining after the minimum capacity \( \lambda_i / \mu \) is assigned to each channel is of great importance. As \( D_n \rightarrow 0 \), the average delay must grow arbitrarily large. In this range, the critical parameters become \( \rho \) and \( \bar{n} \), where \( \rho = \gamma / \mu C \) is the ratio of the rate \( \gamma / \mu \) at which bits enter the network to the rate \( C \) at which the net can handle bits and \( \bar{n} = \lambda / \gamma \), where \( \lambda = \sum \lambda_i \) is the total rate at which packets flow within the net. The quantity \( \rho \) represents a dimensionless form of network "load" whereas \( \bar{n} \) is easily shown to represent the average path length for a packet.

As the load \( \rho \) approaches \( 1 / \bar{n} \), the delay \( T \) grows very quickly, and this point \( \rho = 1 / \bar{n} \) represents the maximum load which the network can support. If capacities are assigned optimally, all channels saturate simultaneously at this point. This formulation \( \bar{n} \) is a design parameter which depends upon the topology and the routing procedure, while \( \rho \) is a parameter which depends upon the input rate and the total capacity of the network.

In studying the ARPANET, a closer representation of the actual tariffs for high speed telephone data channels used in that network was provided by setting \( D = \sum d_i C_i \) where \( 0 \leq \alpha \leq 1 \). This approach requires the solution of a non-linear equation by numerical techniques. On solving the equation, it can be shown that the packet delay \( T \) varies insignificantly with \( \alpha \) for \( 0.3 \leq \alpha \leq 1 \). This indicates that the closed form solution discussed earlier with \( \alpha = 1 \) is a reasonable approximation to the more difficult non-linear problem. These continuous capacity studies have application to general network studies (e.g., satellite communications) and are under continued investigation.

In practice, the selection of channel capacities must be made from a small finite set. Although some theoretical work has been done in this case by approximating the discrete cost-capacity functions by continuous ones, much remains to be done. Because of the discrete capacities and the time varying nature of network traffic, it is not generally possible to match channel capacities to the anticipated flows within the channels. If this were possible, all channels would saturate at the same externally applied load. Instead, capacities are assigned on the basis of reasonable estimates of average or peak traffic flows. It is the responsibility of the routing procedure to permit the traffic to adapt to the available capacity. Often two

IMP sites will engage in heavy communication and thus saturate one or more critical network cutsets. In such cases, the routing will not be able to send additional flow across these cuts. The network will therefore experience "premature" saturation in one or a small set of channels leading to the threshold behavior described earlier.

**DISCUSSION**

A major conclusion from our experience in network design is that message switched networks of the ARPA type are no longer difficult to specify. They may be implemented straightforwardly from the specifications; they can be less expensive than other currently available technical approaches; they perform remarkably well as a communication system for interconnecting time-sharing and batch processing computers and can be adapted to directly handle teletypes, displays and many other kinds of terminal devices and data processing equipment.

The principal tools available for the design of networks are analysis, simulation, heuristic procedures, and experimentation. Analysis, simulation and heuristics have been the mainstays of the work on modeling and topological optimization while simulation, heuristic procedures and experimental techniques have been the major tools for the actual network implementation. Experience has shown that all of these methods are useful while none are all powerful. The most valuable approach has been the simultaneous use of several of these tools.

Each approach has room for considerable improvement. The analysis efforts have not yet yielded results in many important areas such as routing. However, for prediction of delay, this approach leads to a simple threshold model which is both accurate and understandable. Heuristic procedures all suffer from the problem that it is presently unclear how to select appropriate heuristics. It has been the innovative use of computers and analysis that has made the approach work well. For designing networks with no more than a few hundred IMPs, present heuristics appear adequate but a good deal of additional work is required for networks of greater size. Simulation is a well developed tool that is both expensive to apply and limited in the overall understanding that it yields. For these reasons, simulation appears to be most useful only in validating models, and in assisting in detailed design decisions such as the number of buffers that an IMP should contain. As the size of networks continues to grow, it appears that simulation will become virtually useless as a total design tool. The ultimate standard by which all models and

\* Of course the tariffs reflect the discrete nature of available channels. The use of the exponent \( \alpha \) provides a continuous fit to the discrete cost function. For the ARPANET, \( \alpha \approx 2.8 \).
conclusions can be tested is experimentation. Experimentation with the actual network is conceptually relatively straightforward and very useful. Although, experiments are often logistically difficult to perform, they can provide an easy means for testing models, heuristics and design parameters.

The outstanding design problems currently facing the network designer are to specify and determine the properties of the routing, flow control and topological structure for large networks. This specification must make full use of a wide variety of circuit options. Preliminary studies indicate that initially, the most fruitful approaches will be based on the partitioning of the network into regions, or equivalently, constructing a large network by connecting a number of regional networks. To send a message, a Host would specify both the destination region and the destination IMP in that region. No detailed implementation of a large network has yet been specified but early studies of their properties indicate that factors such as cost, throughput, delay and reliability are similar to those of the present ARPANET, if the ARPA technology is used.9

Techniques applicable to the design of large networks are presently under intensive study. These techniques appear to split into the same four categories as small network design but approaches may differ significantly. For example, large nets are likely to demand the placement of high bandwidth circuits at certain key locations in the topology to concentrate flow. These circuits will require the development of a high speed IMP to connect them into the net. It is likely that this high speed IMP will have the structure of a high speed multiplexor, and may require several cooperating processors to obtain the needed computer power for the job. Flow control strategies for large networks seem to extrapolate nicely from small network strategies if each region in the large network is viewed as a node in a smaller network. However, this area will require additional study as will the problem of specifying effective adaptive routing mechanisms. Recent efforts indicate that efficient practical schemes for small networks will soon be available. These schemes seem to be applicable for adaptive routing and flow control in networks constructed from regional subnetworks. The development of practical algorithms to handle routing and flow control is still an art rather than a science. Simulation is useful for studying the properties of a given heuristic, but intuition still plays a dominant role in the system design.

Several open questions in network design presently are: (1) what structure should a high bandwidth IMP have; (2) how can full use be made of a variety of high bandwidth circuits; (3) how should large networks be partitioned for both effective design and operation; and (4) what operational procedures should large networks follow? Much work has already been done in these areas but much more remains to be done. We expect substantial progress to be achieved in the next few years, and accordingly, the increased understanding of the properties of message switched networks of all sizes.

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