Extremal statistics in computer simulation of digital communication systems*

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INTRODUCTION

With the advent of the digital computer it is becoming more and more common to simulate the operation of rather sophisticated communication systems on the computer. The performance of systems under various types of operating conditions may be evaluated quite readily and economically prior to actual field usage.

The average error rate serves as a very common measure of performance for digital communication systems with a probability of error of less than $10^{-5}$ a desirable goal in most system design. Such extremely low error rates pose a real measurement problem, however. Generally with Monte Carlo simulation techniques used one would require data samples of the order of at least 10 times the reciprocal of the error probability to make valid performance estimates, leading to costly and time-consuming simulation runs.

The question of more efficient estimation of low error probabilities in communication system simulation is thus an extremely important one. We report here on encouraging results indicating that the methods of Extremal Statistics may reduce the data requirements in many simulation experiments by at least an order of magnitude.

Major applications of the field of Extremal Statistics have heretofore been made primarily to such areas as Flood Control, Structural design, meteorology, etc. It is only relatively recently that applications to communications have begun to be made, with primary emphasis thus far on the analysis of data obtained from existing systems. Thus, use has been made, in analyzing these data, of special plotting paper developed by Gumbel. Our approach has differed in assuming from the beginning that all calculations were to be made by a high speed computer, that time was of the essence, and that we were interested in applying the theory to the simulation of broad classes of systems.

Extremal statistics is, as the name implies, concerned with the statistics of the extrema—maxima or minima—of random variables. As such it deals with the occurrence of rare events, exactly the problem encountered in simulating low error rate communication systems. It is found that asymptotically (i.e., very large sample numbers of the random variable under study) many of the most common probability distributions follow a simple exponential law when expanded about an arbitrary point on their tails. Thus, the probability of exceeding a specified value or threshold $x_0$ assumes asymptotically the form

$$P_e = P(x > x_0) \frac{1}{n} e^{-\alpha_n(x-x_0)}$$

The number $n$ represents the number of samples used with $\alpha_n$ and $u_n$ constants, depending on $n$, and the actual distribution of the random variable. In particular, the probability of exceeding $u$ is

$$P_e = \frac{1}{n}$$

providing another definition of $u$. Figure 1, for an arbitrary probability density function $f(x)$, shows equation (1) graphically.

The gaussian (normal), Rayleigh, exponential, and Laplacian distributions are among the examples of the asymptotically exponential distributions. All of these distributions may be approximated by Equation (1) in the vicinity of $u$. How far from the vicinity of $u$ one may move depends of course on the actual underlying distribution and the particular point (u) one expands about. As an example Figure 2 compares the exponential approximation to the actual probability of exceedance of $x$, $P_e$, for a gaussian density function. Here $n$ has been arbitrarily chosen as 100. The actual probability $P_e$ and its exponential approximation are then matched at $P_e = 10^{-2}$. It is readily shown that the point $u_n$ about which one expands, is 2.32, and $\alpha_n = 2.68$.

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Comparing the probability of exceedance \( P_e \) for the actual Gaussian and its exponential approximation, as plotted in Figure 2, it is apparent that the two are within 25% of one another at \( P_e = 10^{-3} \) and differ by 50% at \( P_e = 10^{-4} \).

This then points up the significance of the extremal statistics approach: if one is interested in estimating small probabilities of error, say of the order of \( 10^{-3} \) or \( 10^{-4} \), it may be possible instead to first estimate much higher probabilities, say \( 10^{-2} \) in the example of Figure 2. If the exponential approximation is valid one should then be able to extrapolate down to the desired probability. Instead of the usual number of samples required to estimate \( P_e \), say \( 10/P_e \), one can then work with a much smaller number \( n \).

There is of course one major problem, however. Since the underlying density function \( f(x) \) is in general unknown, or difficult to evaluate in the complex systems of interest to us, the two parameters \( \alpha_n \) and \( \mu_n \) are unknown as well, and must be estimated. In the next section we discuss various ways of estimating \( \alpha_n \) and \( \mu_n \), and results of computer runs for two simple density functions, the Gaussian and the exponential. The results are quite encouraging: even with additional samples needed to estimate \( \alpha_n \) and \( \mu_n \), one can still save at least an order of magnitude in the total number of samples required to estimate a given probability of error the traditional way.

In the final section, we discuss the computer simulation of two specific feedback communications systems for which probabilities of error have been estimated quite successfully using extremal statistics. (One of these systems is an example of one for which actual calculations or probabilities of error are quite difficult to make. In the example shown only bounds on the error have been obtained and the simulation results check these quite closely.)

**Estimation of extremal parameters**

We discuss in this section the use of extremal statistics to estimate small probabilities of error in the case of two known distributions, the exponential and the Gaussian. The problem here is twofold: to first estimate the extremal parameters \( \alpha_n \) and \( \mu_n \), then to determine, using these estimates, how well the actual probabilities at the tails are estimated.

The exponential density function normalized to unit variance is given by

\[
f(x) = e^{-\alpha_n \mu_n} \]

\[
\frac{1}{\alpha_n} = 10^{-2}
\]

Both the exponential density function, again normalized to unit variance, is of course given by

\[
f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}
\]

One would expect rather good estimates of the probability at the tail for the exponential density function since it is already in the asymptotic form of equation (1). In the Gaussian case, as pointed out in the previous section and as illustrated for one case in Figure 2, it is theoretically possible to extrapolate as much as two orders of magnitude away from the starting point \( 1/n \) before the quadratic exponential behavior of the gaussian function takes over and produces significant deviations away from the linear exponential behavior of extremal statistics.
The actual experimental behavior of the exponential approximation depends critically on the estimation of the two parameters \( \alpha \) and \( u \). To determine these we use the fact, as demonstrated by Gumbel,\(^1\) that they are intimately connected to the asymptotic statistics of the extrema (maxima) of the random variable \( x \). Specifically, if one generates \( n \) independent samples of \( x \) the probability density function of the largest (maximum), \( x_{m} \), of the \( n \) samples is asymptotically \( (n \to \infty) \) given by

\[
P_{n} = \alpha e^{\alpha} \left[ -\left( y - e^{-\alpha} \right) \right]
\]

Equation (4) is found to be valid for a wide class of density functions possessing exponential behavior at the density functions possessing exponential behavior at the tails, with the exponential, gaussian, and Rayleigh functions typical examples.

From Equation (4) it is readily shown that \( u_{n} (n \to \infty) \) is a measure of the mode of \( p_{n}(x_{m}) \), while \( \frac{1}{n} (n \to \infty) \) is a measure of the dispersion. Specifically, one finds, using Equation (4), that

\[
\frac{1}{\alpha} = \sqrt{\frac{6}{\pi}} \sigma_{m}
\]

and

\[
u = E(x_{m}) - \frac{\gamma}{\alpha}
\]

Here \( \sigma_{m} \) is the standard deviation of the maximum (extremal) values, \( x_{m} \), of \( x, E(x_{m}) \) the expected value of these maxima, and \( \gamma = 0.5772 \) is just Euler's constant.

It is thus apparent, that to estimate \( \alpha_{n} \) and \( u_{n} \) one must first ensure \( n > 1 \) (This is why Equation (1) is applicable to the tails of density functions, where \( P_{e} < 1 \)), and then generate sufficient samples of the random variable \( x \) under test to measure their statistical properties. If \( N \) samples of the largest value of \( x \) in a group of \( n \) are to be made available this implies repeating the experiment \( nN \) times in all. It is the total number \( Nn \) that is to be compared to the usual number \( 10P_{e} \).

From the form of Equations (5) and (6) one would expect that for \( n \) and \( N \) large enough, good approximations to \( \alpha_{n} \) and \( u_{n} \) would be obtained by averaging appropriately over the \( N \) samples of the maxima available. As noted later this was in fact found to be the simplest and most accurate procedure in actual experimentation with the computer. This estimation procedure is portrayed in Figure 3. There, as an example, \( n = 10^4 \) and \( N = 10 \) are chosen. The total number of independent samples or repeats of the computer simulation involved would thus be \( nN = 10^4 \).

The resultant output samples would be grouped into \( N = 10 \) groups of \( n = 10^3 \) samples each. The largest sample, \( x_{i} \), in each group would then provide \( N = 10 \) samples with which to estimate \( \alpha_{n} \) and \( u_{n} \).

\[
\text{Figure 3 - Estimation of } \alpha \text{ and } u
\]

There is a tradeoff possible between \( n \) and \( N \), given the fixed number of repetitions \( nN \). Thus decreasing \( n \) decreases the range over which one would theoretically expect the asymptotic exponential approximation to hold (assuming perfect knowledge of \( \alpha_{n} \) and \( u_{n} \)), but allows better estimation of \( \alpha_{n} \) and \( u_{n} \) as \( N \) increases. Some analysis of the optimum choice of \( n \) and \( N \) has been carried in a recently completed doctoral thesis.\(^4\)

In the actual computer simulations carried out \( n \) was taken as \( 500, N = 20 \), so that a total of \( 10,000 \) actual repetitions of the different experiments tried were performed. Normally this would provide relatively accurate estimation of probabilities of error as low as \( 10^{-3} \). We were interested in extending the estimation to \( 10^{-4} \) and \( 10^{-5} \).

As noted previously, an obvious initial estimate for \( \alpha \) is to replace \( \alpha_{n} \) in Equation (5) by the sample standard deviation \( s \), using the \( N = 20 \) samples available of the extrema. Similarly a first estimate for \( u \) is to replace \( E(x_{m}) \) in Equation (6) by the sample mean \( \bar{x}_{m} \) (again using the 20 extremum samples). (These are the procedures suggested in Figure 3.) Although the sample standard deviation is in general a rather poor estimator of the statistical standard deviation \( \text{var} (s) = 2\sigma_{m}^{2} / N \) for gaussian statistics] the experimental results obtained were surprisingly good for both the exponential and gaussian distributions: the estimates of \( \alpha \) came within 10% of the true values in both cases. Similarly the estimates of \( u \) came within 3% of the true values, well within the confidence limits set by using the sample mean estimates.
These initial estimates were compared experimentally with several other approaches:

1. Initial estimates obtained using approximate solutions of the maximum likelihood estimates of \( \alpha \) and \( u \) (these involve fourth sample moments).
2. Initial estimates obtained by an approximation procedure attributable to Kimball.\(^1\)
3. Initial estimates obtained by using the first and second extremal values. (One might expect that the second extremal values should provide some information on \( \alpha \) or \( u \), and are available in a simulation run anyway.)

Using Equation (4) the maximum likelihood estimates of \( \alpha \) and \( u \) are found to be given in terms of the \( N \) extreme (maximum sample values \( x_j, j = 1 \ldots N \), by solution of the following two equations:\(^1\)

\[
\frac{N}{\alpha} = \sum_{j=1}^{N} x_j [1 - e^{-\gamma j}] \quad (7)
\]

\[
N = \sum_{j=1}^{N} e^{-\gamma j} \quad (8)
\]

Here \( \gamma_j = \alpha(x_j - u) \).

These equations cannot be solved specifically for \( \alpha \) and \( u \), but may be either iterated or approximated in various ways.

The two equations may be combined to eliminate \( u \), resulting in the following equation in \( \alpha \) alone:

\[
1 = \frac{\sum_{j=1}^{N} [x_j - \bar{x}_m] e^{-\alpha(x_j - \bar{x}_m)}}{\sum_{j=1}^{N} e^{-\alpha(x_j - \bar{x}_m)}} \quad (9)
\]

Here \( \bar{x}_m = \frac{1}{N} \sum_{j=1}^{N} x_j \) is the sample average of the \( N \) extrema (maxima). Equation (9) lends itself readily to both approximation and iteration. (Note that since \( \frac{1}{\alpha} \) is positive this equation indicates that values of \( x_j \) less than \( \bar{x}_m \) will occur more frequently than those greater than \( \bar{x}_m \). This is of course due to the asymptotically exponential character of the distribution of the extrema.)

An approximation of \( \frac{1}{\alpha} \) suitable for simple calculation is obtained by expanding Equation (9) in an infinite series, retaining the first few terms, and reordering the resultant equation. This provides the following approximate estimate of \( \alpha \):

\[
\alpha^2 = \frac{2 s^2}{u_4} \left[ \sqrt{1 + \frac{2 u_4}{s^2}} - 1 \right] \quad (10)
\]

Here \( s^2 = \frac{1}{N} \sum_{j=1}^{N} (x_j - \bar{x}_m)^2 \) is the sample variance of the \( N \) extremal values, and \( u_4 = \sum_{j=1}^{N} (x_j - \bar{x})^4 \) is the sample fourth moment. (It is assumed, in deriving Equation (10), that the third sample moment \( \sum (x_j - \bar{x})^3 \) is zero. Although true for symmetrical distributions such as the gaussian, this moment is, on the average, negative for the extremal statistics. The assumption simplifies the equation considerably, however.)

The maximum likelihood Equations (7) and (8) for \( \alpha \) and \( u \), were iterated several times using as first estimates the different approximations noted earlier:

1. The asymptotic Equations (5) and (6) for \( \alpha \) and \( u \), with the standard deviation \( \sigma_m \) replaced by the sample deviation \( s \) and the expected value \( E(x_m) \) replaced by the sample mean \( \bar{x}_m \).
2. Equation (10) for \( \alpha \).
3. A technique attributable to Kimball.\(^1\)

Interestingly it was found by computer experimentation that Gumbel's asymptotic estimates of Equations (5) and (6) were both the simplest and fastest to implement on the computer, and in all cases tested also came closest to the true values on the initial try. (All four methods produced very nearly the same results after iteration.)

The first estimate using Equation (10) was also fairly close to the expected value of \( \alpha \) on all experiments performed. The Kimball method noted was, however, quite inaccurate.

Using the best estimates of \( \alpha \) and \( u \) obtained, extremal statistics were in turn used to estimate error probabilities for both the gaussian and exponential distributions. Results obtained compared favorably with those predicted theoretically using extremal statistics: with the total number of samples used, \( nN = 10,000 \), probabilities in the vicinity of \( 10^{-4} \) were estimated within 20%, while for probabilities as low as \( 3 \times 10^{-5} \) the estimates were within 60%. (For low probabilities such as these, 100% variations still provide significant information. These compare with the more common statistical procedure of using 10,000 samples to estimate probabilities of the order of \( 10^{-3} \), or at best perhaps \( 5 \times 10^{-4} \).)

The usefulness of the extremal statistics approach for estimating small probabilities is further seen in comparing confidence intervals with these obtained for the more common procedures. Since in the extremal statistics case it is the estimation of the parameter \( \alpha \) that is more subject to large variations for \( N = 20 \) samples one may assume the parameter \( u \) fairly well estimated on the basis of the same 20 samples. Confidence intervals on the probability of error
curves may then be set using the standard deviation of the estimate of \( \alpha \). For the gaussian distribution and \( n = 500 \) samples it is then found that the \( \pm 1 \sigma \) confidence interval for \( \alpha \) corresponds to the following bounds on the measured probability:

<table>
<thead>
<tr>
<th>Theoretical probability</th>
<th>Upper bound</th>
<th>Lower bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^{-3}</td>
<td>1.1 \times 10^{-5}</td>
<td>0.7 \times 10^{-5}</td>
</tr>
<tr>
<td>10^{-4}</td>
<td>2 \times 10^{-4}</td>
<td>0.5 \times 10^{-4}</td>
</tr>
<tr>
<td>10^{-5}</td>
<td>5 \times 10^{-5}</td>
<td>0.4 \times 10^{-5}</td>
</tr>
</tbody>
</table>

Although the usual Tschebychef inequality confidence intervals (based on estimating probabilities by relative frequencies of occurrence) give roughly the same bounds at 10^{-3}, the lower bound deteriorates rapidly, below this probability, going to 0 at 10^{-4} samples available. One can of course say nothing, except possibly be extrapolation, of probabilities in the vicinity of 10^{-5} using the relative frequency approach.

**Application to feedback communication systems**

In the previous section we have discussed methods of estimation of the two extremal parameters \( \alpha_n \) and \( \alpha_u \), using the known gaussian and exponential density functions to determine the most efficient ways of accurately estimating these parameters.

We now discuss the application of the method of extremal statistics to the simulation of digital feedback communication systems. Two systems of considerable interest currently have been investigated. One is a binary signalling system, the other an M-ary PAM type system, with the information transmitted as one of \( M \) possible amplitude levels. In both cases white gaussian noise is assumed added during forward transmission (this could be thermal noise introduced at the receiving antenna, front end receiver noise, or a combination of the two in general). The feedback path is assumed error free. Errors in signal detection due to the noise may occur and it is desired to estimate the probability of error in both systems.

Both systems may have potential usefulness in space-ground communications. In both cases the information is to be transmitted from space to earth. The forward transmission path is thus limited in power. It is assumed a feedback path from ground to vehicle with much larger power capability is available, so that the effects of noise may be neglected over this path. (Further computer simulation is planned to investigate the effect of noise on the feedback path, as well as other signal disturbances such as fading for example.) Such feedback systems hold great interest in the statistical communications field currently because of their expected high performance (low probability of error) in noise with a minimum of coding effort required.

The binary system investigated represents the application of sequential decision theory of statistics to statistical communications. The operation at the receiver may be visualized by referring to Figure 4. Here \( v \) represents a received sample of the composite signal plus noise. As an example if one binary signal has a received amplitude of \(+A\) volts, while the other is \(-A\) volts (this is the example of bipolar signalling), the resultant probability density functions \( f_1(v) \) and \( f_2(v) \), respectively, appear in Figure 4.

![Figure 4 - Received signal space, binary feedback](image)

In normal binary signalling one would declare signal 1 received if the received signal plus noise sample were to exceed 0, and signal 2 received if the received signal plus noise fell below zero. For small signal-to-noise ratios (SNR), as normally encountered in deep space-ground communication, the resultant probability of error would be intolerably high. To improve the performance a null zone of width \( \pm a \) about 0 is set up. If the composite received signal falls in this region a decision is deferred and the transmitter asked, via the feedback path, to repeat the signal. The first composite signal sample is then stored and added to the second received sample after transmitter repetition. The two combined received samples are then tested and a decision made only if the sum exceeds \( +a \) or falls below \(-a\). A third repeat is requested if the sum again falls in the null zone.

This procedure is repeated until the combined received samples fall outside the null zone. (To prevent the system from cycling indefinitely or for too long a time, the number of transmissions may be truncated after a specified interval and a definite decision made.) Such a sequential procedure with a statistically variable number of transmissions may be shown to asymptotically provide 50% decrease in average transmission time over an equivalent system without feedback with a fixed number of repeats, this for the same SNR and probability of error.

Although the scheme outlined is relatively simple to describe the analysis is quite complex, since the received signals summed are these falling in the null zone only. The statistical behavior of the composite random variable representing the sum of such received signals is difficult to determine and the resultant
probability of error not easily found. In fact asymptotic results only for the probability of error are available; those for small SNR and a large average number of transmissions, or those for high SNR and very small numbers of transmissions. The region in between can only be estimated by extrapolation or by bounding techniques. Computer simulation of such a system is, however, simple to perform. For small probabilities of error, however, the computer time involved can become quite large so that extremal statistics is an obvious answer.

The methods of extremal statistics were therefore used, in a computer simulation of this system, to estimate the probability of error. \( nN = 10,000 \) total repeats were used in each simulation, \( N = 20 \) for estimating \( a \) and \( u \), \( n = 500 \) for the actual determination of the probabilities. Results are shown in Figure 5. Note that the experimental (simulation) points follow quite closely an approximate performance line obtained by extrapolating from Wald's asymptotic results, valid for large numbers of transmissions and small probabilities of error. The curves of Figure 5 are for a SNR of \( \frac{1}{2} \). A reference curve, that showing the performance of a one-way system with no feedback and a fixed number of repeats, is included. Note that the two curves differ approximately by a factor of two in the transmission time, as would be expected from Wald's asymptotic results.

The second feedback communications system simulated, for which extremal statistics were used in estimating system performance (probability of error), was one first suggested and analyzed by J. P. Schalkwijk. A simplified block diagram appears in Figure 6. The information to be transmitted consists of one M amplitudes, \( \Theta = \frac{1}{M} j = 1, \ldots, M \) (the amplitudes are normalized to a range of 0 - 1 for convenience). T seconds are available for transmission and it is assumed that N signal transmissions, each lasting \( \frac{T}{N} \) seconds, are made. (Note that this fixed transmission time scheme contrasts with the sequential binary scheme, requiring a variable transmission time, just discussed.)

On the \( k^{th} \) transmission the signal transmitted is of the form

\[
s_k = g_k (\Theta - \hat{\Theta}_{k-1})
\]

with \( g_k \) a variable but known gain factor, and \( \hat{\Theta}_{k-1} \) the receiver's maximum likelihood estimate of \( \Theta \), based on the first (\( k-1 \)) transmissions. The receiver transmits \( \hat{\Theta}_{k-1} \) to the transmitter via a noiseless feedback path. This system is of the information feedback type, since actual information as to the received signal plus noise is fed back to the transmitter.

![Figure 6 - M-ary information feedback system, N respects](from the collection of the Computer History Museum (www.computerhistory.org))

On each successive transmission the receiver's estimate of the correct signal, \( \Theta \), improves so that \( \hat{\Theta}_k \) approaches \( \Theta \) more closely. After N repeats the nearest \( \hat{\Theta}_N \) to \( \Theta_n \) is chosen as the correct signal. It may then be shown\(^{7,8} \) that the probability of error goes essentially as

\[
P_e \sim \exp \left[ -\frac{3}{2} e^{(C-R)T} \right]
\]

for large \( T \), and \( R < C \). Here the gain \( g_k \) of the \( k^{th} \) transmission has been optimized to provide minimum probability of error. The parameter \( R \) is the rate, in bit/second, of signal transmission:
\[ R = \frac{\log_2 M}{T} \text{ bits/sec.} \quad (13) \]

and C is the so-called channel capacity in bits/sec. for this gaussian channel:

\[ C = W \log_2 \left[ 1 + \frac{P_{av}}{nW} \right] \quad (14) \]

Here \( W = \frac{N}{2T} \) is the channel bandwidth, \( P_{av} \) is the average power available from the transmitter, and \( n/2 \) is the white noise spectral density.

(Equation (14) is exactly the channel capacity of the gaussian channel first obtained, using a random coding argument, by Claude Shannon. It represents the maximum rate of transmission, in bits/sec., available for error-free transmission over such a channel. Schalkwijk's scheme is the first specific signalling scheme known to obey Shannon's previously ideal signalling law, and this accounts for the interest stirred up in it.)

A computer simulation of this system was carried out, with the probability of error found using extremal statistics. Here, because of the relative complexity of the system, only a limited amount of simulation could be carried out. Specifically, for \( N = 25 \) repeated signal transmissions and \( R/C = 0.5 \), the probability of error was found to be 0.037, while for 50 signal transmissions, the probability of error was reduced to \( 2.3 \times 10^{-3} \). Both of these numbers compare favorably with results of an exact analysis, possible here, the complexity of the system notwithstanding, because of the additive white gaussian noise assumed and linear operations at both transmitter and receiver. (Had the noise been nonwhite or nongaussian, or had some simple nonlinear operations been included exact analysis would have been out of the question. Yet the simulation complexity would not have been substantially different, pointing out the utility of simulation here.) For the estimation of the probabilities here, 20 extremal values were again used to estimate \( a \) and \( u \), while \( n = 50 \) samples were used in the estimation of probability. The total number of samples per simulation was thus 1,000.

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