HUMAN DECISION MAKING UNDER UNCERTAINTY AND RISK: COMPUTER-BASED EXPERIMENTS AND A HEURISTIC SIMULATION PROGRAM*

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INTRODUCTION

Every organism of higher order has to make decisions of varying importance regularly and frequently in order to survive and to survive efficiently. While the outcome of decision making has been studied extensively by a wide range of different disciplines, the decision making processes themselves have been neglected in comparison. Emphasis has been placed on the normative aspects of human behavior, i.e., how a rational† person or a group of rational persons ought to behave, as distinct from descriptive theories which are to explain and predict actual human behavior. An overwhelming majority of techniques and methods of attack in operations research, management science, industrial mathematics, etc. could be given the label: Normative Decision Theory. The students of mathematical psychology, on the other hand, beginning probably with Lady Lovelace, Bernoulli and Laplace, have been concerned with the behavioral aspects of decision making, i.e., what is being done in certain situations, and why.1

In recent years another approach to understanding human behavior has emerged through the pioneering work of Newell, Simon and their followers.‡ The Complex Information Processing method (CIP) dissects, for example, problem solving activity into elementary building blocks, such as different phases of planning, building of subgoals, utilization of certain heuristics, etc. These information processes and an executive routine are programmed for a digital computer. Comparing the trace of the program with the verbal reports and protocols elicited from subjects may indicate the level of completeness of the theory.

*This terminology is not quite uniform. Most authors in statistical decision theory and economics have adopted the convention that in a situation of uncertainty there is no a priori information about the system at hand, consequently probability distributions of various alternative outcomes cannot be objectively specified. In cases involving risk, however, it is assumed that, relying on prior events of identical character, these frequency distributions are available. In the present work we have not distinguished between these two cases and described the psychological state of the subjects as that under uncertainty. The element of risk is represented by the subjects' financial involvement in the outcome of the experiments. The adopted terminology and the distinction made here may be rather useful with experiments which aim at discovering correlation between subjects' behavior and varying payoff matrices while the stochastic components of the environment stay stationary.

†A "rational" person is (a) completely informed, (b) infinitely sensitive, and (c) can formulate his problem in an optimum manner.

‡The basic philosophy of this approach has been described in detail, e.g., in reference 2.
Simon and Newell make the following three underlying propositions in this regard:

1. A science of information processing can be constructed that is substantially independent of the specific properties of particular information-processing mechanisms [meaning, for example, which particular language is used on which particular machine—N.V.F.].

2. Human thinking can be explained in information-processing terms without waiting for a theory of the underlying neurological mechanisms.

3. Information-processing theories of human thinking can be formulated in computer programming languages, and can be tested by simulating the predicted behavior with computers.

Special problem-oriented list-processing languages, such as IPL-V, LISP, COMIT, etc., have been developed to enhance the symbol-manipulating capability of computers. Projects ranging from game-playing through theorem-proving and concept learning have proved that it is possible to find a sufficient theory, embedded in a computer program, that gives satisfactory explanation for different cognitive activities. Perhaps the most significant, but as yet incomplete, achievement is the discovery of universal features in human problem solving, that are independent of the particular task at hand. Continuing this line of reasoning, it remains to be seen how a successful chess playing program can make use of its skill in learning, e.g., bridge, or how a Russian-into-English mechanical translator would be in a superior position in adapting itself to, say, French-into-English translating, as compared with a linguistically naive machine.

Information processing models exist for certain decision making tasks, such as Feldman’s program, which simulates human behavior in the binary choice situation and Clarkson’s program which makes trust fund investments.

The present work aims at a particular type of decision making. The project has two phases. First, a series of computer based experiments were designed and carried out. In these, human subjects were placed in quantitative task environments and their verbal behavior was observed and recorded after they had been asked to think aloud. The first part of the paper lists the research objectives, outlines the task environment and discusses the “design principles” resulting in the particular organization of the experiments.

The other phase of the project, dealt with in the second half of the paper, describes the subjects’ behavior and attempts to establish a theory for the above behavior. Following the lines of the CIP approach, we formalized in terms of flow charts the subjects’ internal representation of the environment, and the rules and methods they have employed in their organized, problem-oriented activity. The theory was then tested. The particular mechanisms, the elementary information processes, specified by the flow charts can be either hand simulated or, in view of their highly complex nature, programmed for a digital computer and the subjects’ behavior compared with the computer performance. It has been found that the theory under discussion not only is a satisfactory explanation of average human behavior, which achievement is the usual goal of most endeavors in psychology, but can also describe certain characteristic features of individual behavior. In order to be able to do so we have departed to some extent from the totally deterministic viewpoint of the logic theorist or the general problem solver and have incorporated certain stochastic components of limited importance in the framework of this theory to bridge over the gap of our ignorance about intraperson inconsistencies and interperson differences. This fact and the manner in which an individual is initially characterized for the program (cf. executive program) render the theory solely phenomenological. This, however, has been the contention of researchers at this level of explanation.

THE RESEARCH OBJECTIVES

The project has three basic research objectives of methodological and theoretical implications:

1. To develop techniques for organizing complex decision making areas in terms of their internal structure so as to make subjects exhibit specific aspects of behavior.

2. To test the applicability to control tasks of the CIP method and to see the limits of respective theories embodied in the form of computer programs.

3. To further our understanding on human decision making under uncertainty and risk.
The following research objectives of secondary importance also seem to be appropriate:

4. To assess the capability of man confronted with a complex quantitative problem situation;
5. To describe the cognitive and affective behavior of humans trying to gain understanding of partially confounded causal relationships;
6. To explore the effects on search behavior of task environments of different internal structure;
7. To investigate the extent and, possibly, the motivational background of individual differences in decision making behavior;
8. To study the efficacy of different subjective representational languages in problem solving;
9. To identify patterns of response to 'regular' and 'unexpected' changes in the environment; and so on.

THE ORGANIZATION OF THE EXPERIMENTS

The psychological study of decision making is often handicapped either by the restricted task environment or by the significant but irrelevant factors possibly affecting the outcome of the experiments which are carried out under "nonlaboratory", i.e., not fully controlled, conditions. These difficulties are hoped to have been at least partially overcome by the simulated realism of computer-based experimentation.

Computer-based experiments may aid at man-machine relationships per se or use different modes of interaction as flexible tools to study the behavior of humans in a variety of well-determined environments. The present work was intended to cover both areas with a definite emphasis on the latter.

The experimental setup was as follows:

The subject was situated in front of a computer console which represented his input/output medium to a complex continuous technological process. The nature of and the laws regulating the process were not known to the subject. At his disposal there was a set of control variables, \( \{x\} = x_1, x_2, \ldots, x_m \); further he could observe a set of state variables, \( \{y\} = y_1, y_2, \ldots, y_n \). Each of these state variables was associated with a cost variable through a cost function, \( c_i = h(y_i) \), in a simple and well-defined way. (This relationship was, however, not given explicitly to the subject.) The task of the subject was to run the system at minimum overall cost, \( \Sigma c_i \rightarrow \text{Min!} \), over some period of time. (See Fig. 1)

![Figure 1. Block diagram of the organization of the experiments.](From the collection of the Computer History Museum (www.computerhistory.org))
the subjects were asked to think aloud. Their verbal behavior was tape-recorded and the transcripts of these protocols, together with the computer input/output, constitute the experimental data.

From a utilitarian point of view the advantages of this setup are manifold: the experimenter can vary the internal structure of the “black box” process and the cost functions at will to suit his particular needs; information-processing errors are virtually excluded; the quantitative nature of the environment renders the stimuli and the responses direct and unambiguous; experimental replications are easy and inexpensive to perform. Further, a whole gamut of experiments can be conducted, and are in fact planned for future studies, on game learning, game playing, coalition formation if certain subsets of control variables are assigned to different players. Experience can also be gained on how human operators adapt themselves to different real-life control tasks, etc.

PRINCIPLES OF THE PROCESS DESIGN

The experimenter in psychology naturally would like his subjects to exhibit as wide a spectrum of relevant behavior as the physical and conceptual limitations of the experiment allow. He also wants to see what correlations there are between certain changes in the task environment and the subject’s behavior so that his previous goal can be achieved.

Unfortunately there is nothing like the well-formulated statistical theory of experimental design to be applied to meet these problems. The designer has to resort to “analog computations,” i.e., he either imagines himself into the subjects’ role or tries out different tentative tasks with subjects to see whether their gross behavior in fact tends to be of the type he had anticipated. As we have mentioned before, the process is represented by a set of mathematical relations with some noise superimposed upon it. Unless we want the subject to come to a high degree of confusion, the average noise level should apparently be adjusted to be about 1/10—1/20 of the average state variable level. The periodicity of noise should also be low enough not to be noticed. Instead of pseudo-random numbers we have used algebraic type noise generators, examples of which are as follows:

1. $A_i \pmod{B}$, where $A$ and $B$ are relative primes and their magnitude is roughly determined by the required level and tolerated periodicity of noise, respectively; $i$ is the number of times noise has given input values.

2. $C \sin \frac{k \cdot \pi}{D}$, where $C$ and $D$ are constants the values of which are determined again by the required level and tolerated periodicity of noise, respectively; $k$ is the number of times the subject has received outputs.

3. $E \delta_{ks}$, where $E$ and $F$ are constants the values of which are determined again by the required level and tolerated periodicity of noise, and $\delta$ is Kronecker’s delta function. This type of noise is represented by a sudden peak of size $E$ every time $k$ is a multiple of $F$.

4. Some simple functions of $|\Delta x_i^{(i)}| = |x_i^{(i)} - x_i^{(i-1)}|$, i.e., of the absolute value of the change in the $i$th control variable between the $(i-1)$th and $i$th input (step size).

5. Some simple functions of $|x_i^{(1)} - x_i^{(1)}|$, i.e., of the absolute value of the difference between two control variables input concurrently, and different variations and combinations of all the above types.

Types (1) and (2) provide “background noise,” with otherwise no specific role. Type (3) can be used in studying response patterns to sudden “unexpected” changes as distinct from “regular” fluctuations. The last two types of noise can be used for penalizing or rewarding the subject if he is making too big or too small steps or if he deviates from certain regions in the control variable space. Type (4) is equivalent to a short term memory in the system, while type (5) represents a reasonably simple interaction between two control variables.

We can provide the subject with state variables of high variability in the low-cost region or with variables of low variability in the high-cost region. (He can achieve subgoals in comprehension at a certain cost or remain partially in the dark inexpensively.) This could induce the subject to make a distinction between short-term and long-term objectives, which tendency may be important if the payoff is effected according to time-average achievements. Sophisticated persons sometimes would like to suboptimize detached parts of the system even at a certain cost. This can be realized with carefully thought out interaction terms. Other task environments can help us find out the upper limit of the
number of state variables the sum of which is approximately constant, regardless of the control variables, and the subject can still detect this fact within some reasonable time. Or, ascertain how long it takes a subject to find out that certain state variables are purely noise functions, which control variables interact, what the most reasonable search behavior is in terms of step size, etc.

In the course of setting up the experiment we found a few golden rules of thumb, following which a fairly systematic approach developed and assumed much of the role of a fool-proof theory of "experimental design." These are: it is good to have a few but not too many local minima; the absolute minimum should not be either in the middle or at the limits of the control variable ranges; interaction between control variables should be noticeable but not predominant. In general, it is advantageous to limit the range of all control variables identically. This range should be big enough to allow sufficient exploration but not too big so that subjects can examine and possibly even memorize important features of the whole range. In this study $0 \leq x \leq 50$ and $0 < y < 1000$ for all $x$'s and $y$'s.

In order to avoid the possibility of the subject's drawing sketches of two or three-dimensional response patterns on the one hand, and overcomplicated multidimensional searches on the other, task environments with three control and three state variables were uniformly selected.

For the sake of simplicity, in this series of experiments linear cost functions of the type $c_i = \alpha_i + \beta y_i$ were used.

Four exemplary task environments are described in Appendix I.

THE SUBJECTS' BEHAVIOR

In view of the relative complexity of the task environment one would not expect every subject to behave identically, nor even the same person to take identical courses of action in the same situation at different times. The crucial question that has to be answered by students in behavioral sciences is whether there are significant characteristics of demonstrable generality that are common either for everybody or, at least, for sizable groups of human beings. If we are able to discover such characteristics of all the major facets of human behavior then a causal psychological theory of great expiatory and predictive power is, figuratively speaking, just "around the corner." As has been mentioned before, the solving of fairly well-defined problems by humans is a type of activity that does reveal certain general characteristics and the General Problem Solver, which simulates human problem solving on a computer, is a successful vehicle for its theory.

A theory of decision making can claim a higher degree of validity if it incorporates, besides cognitive processes, also some phenomena of affective behavior, such as aspiration, interest, curiosity, etc. Many, although far from all, subjects can be induced to express in words their motives, even the ones they, themselves, regard as childish or nonsensical. As a result of our continuous effort, subjects did stay fairly communicative. Consequently, a detailed analysis of the verbal reports, the extraction of essential elements and the systematic ordering and classification of the motivated actions were not negligibly small tasks.

So far 16 subjects' behavior has been studied. Some characteristics of the findings are summarized in Appendix II. Appendix III contains fairly detailed excerpts from a specimen protocol. Although a sample of this size would not allow us to draw minute conclusions of high statistical significance, a great amount of information was collected on a certain kind of human behavior. It is believed that the following, nonquantitative discussion, relying on experimental evidence, represents facts of reasonable generality.

The Initial Search

The initial search behavior of subjects can be divided into two broad categories. Some of them tend to explore the task domain, i.e., the range of control variables, in an allegedly 'random' fashion. They try to generate as many different combinations of numerical values as possible. However, these attempts obviously reveal some sort of basic pattern, which the subjects, behaps unconsciously, use as a guide in the search. The fundamental human discomfort in experiencing or producing irregularity can be easily detected in these cases. The net result of this type of search is not much different from that of the other category of subjects who take pride in being

*As one subject (of what sex?) said: "I can see small particles swirling around in a big blue room, hitting each other and the walls. The faster they fly the higher values these $C$'s take."
"systematic." The search for structure, so inherent in human behavior, is modulated by preformulated ideas, such as

"Let me first see all the corners," or
"I'll explore the center thoroughly," or
"I want to go along this line at first," or
"Everything is somewhat symmetric," etc.

Subjects of varying background and apparent capabilities have shown such Einstellung effects. As Luchins\(^\text{16}\) in a different context has noticed, persons of superior intelligence and education develop habituation effects to as great as or even greater degree than do persons of low educational level and IQ. As possible motives he lists for example the educated man's desire to generalize and also his contempt of childish or ostensibly repetitive tasks. In contrast with Luchins' findings, our subjects did not exhibit a mechanistic blind attitude toward the problem for any significant length of time, particularly not so after the initial search just described. The information feedback from the outcome of individual trials seemed to have an effective influence on the search behavior.

The initial search goes on with both types of subjects until they form the first hypothesis about the organization of the task. This event can happen quite suddenly, "out of the blue," or may be preceded by a long and thoughtful meditation. The subject "cannot help but discover it" or else he may force himself to visualize some sort of model for his problem. As one would expect, not only the type but also the length of the initial search varies from subject to subject within certain limits.

The Hypotheses on, or the Internal Representation of, the Task Environment.

The subjects' approach to the task varied considerably. Each of them understood the problem apparently equally well but interpreted it rather differently. The differences in personality, educational background and, maybe, temporary disposition resulted in somewhat different sets of "intellectual tools." We shall call this set of tools a subject's language of representation and the model built with its help the mental image of the task environment.

Before we discuss our findings in this regard, let us digress briefly and investigate the basis and implications of the above-mentioned concepts.

Man's information-handling capability is, in general, enhanced by thinking in terms of symbolized concepts whether these be elements of a spoken language, pictographs, logical constructs or mathematical relationships. In meeting man's changing needs in problem solving and decision making, sophisticated and powerful methods of attack can coexist with hunches, "intuitions," "insight," \textit{déjà vu} processes. An important part of the learning process consists of improving and replacing conceptual hypotheses, which are formed after processing the received and self-generated information complex. These hypotheses are mentally combined and represent the image of the outside world (\textit{Weltanschauung}). The combination of hypotheses is based on elementary conscious processes, mentioned before, such as remembering, comparing, recognizing, abstracting, logically interpolating and extrapolating, etc.

Man's problem-solving methodology starts from and depends on the language of representation he has formed to comprehend the environment. Pattern perception, verbal learning and concept formation are examples of the interaction between the language or representation and the resulting cognitive behavior.

In trying to perceive the relations of the stimulus environment, humans also try to relate their own behavior to the variations in it.\(^\text{17}\) This fundamental tendency connects the so-called internal and the external structures, and renders the environment "meaningful." If these two structures are fairly homomorphic with a particular language of representation and with a particular task environment, the building of a mental image will be comparatively easy and successful. Similarly, with a problem-oriented computer language the data representations and the information processing instructions are such that we can solve with it a family of problems efficiently. A good problem solver finds good languages of representation, just like a good (multilingual) programmer is able to select the best computer language for his task.

The origins of the hypotheses about the environment can only be partly explained by the Gestaltist argument that the behavior is an inevitable result of some stimulus configuration and that the stimulus forces a specific response. It is also true that responsive behavior relies on the individual's interpretation of the environment. Any new situation to a subject is not a confused, meaningless conglomera-
tion of sensory impressions to which he makes confused, meaningless, uncoordinated and unrelated responses. He brings to the new situation a whole history of experience which he is ready to apply. Each response of the subject we observe is not a *ding an sich* but a meaningful part of his total behavior, which has casual roots both outside and inside.

Consequently, the decision making behavior (a) is systematic and purposive (displaying an "if-then" character), (b) involves some degree of abstraction, and (c) does not depend entirely upon the external environment for its initiation and performance.

Let us now see the essential elements of the major types of representation the subjects have revealed.

The "Mathematical" Language of Representation.

This is the most frequently occurring type (NB: this was also the "true" language of representation), which fact is probably due to the biased nature of our sample. Not every college student subject of ours has, however, had mathematical training at tertiary level. Some were undergraduates in psychology and in fine arts (cf Appendix II).

The state and control variables are thought to be connected by a set of mathematical relations, which are somehow perturbed by extraneous noise. The search behavior can be divided into two, rather distinct stages. First, the subject is only concerned with the overall cost, \( \sum c_i = C \), and its relation to different levels of control variables. In specifying new points in the \( \{ x \} \) space he follows certain rules, which will be described in detail in the next section dealing with the simulation of the behavior. The subject discovers the presence of noise fairly soon and tries to overcome its disturbing effect by averaging operations. Intermediate successes and failures raise and lower his aspiration levels, respectively, and eventually cause him to enter the second stage.

The second stage is a more sophisticated continuation of the first one. The subject tries to discover relations between individual state and control variables. How he does this depends largely on the extent of his mathematical training. We were able to discover more or less distinct versions of hill-climbing, univariate, factorial and random methods. The most frequent approach followed a pattern similar to the one described as direct search by Hooke and Jeeves[18] and employed by, e.g., Flood[19] This fact is quite reasonable, there being no universal analytical technique or computational scheme that will always work in case of highly irregular functions with ridges, temporary plateaus, saddle points, etc.; Hooke and Jeeves themselves incorporated in their method heuristic procedures that "do not guarantee . . . a correct solution . . . ."

These optimizing procedures were seasoned with occasional jumps of 'sudden insight' at times when a new idea has occurred to the subject or when a response pattern started to resemble one already known to him. (This could be a *déjà vu* pattern in the first few minutes.)

The techniques associated with the mathematical language of representation were the most successful ones and the language itself was usually maintained throughout the session unless a sufficiently high level of frustration removed the subject from it into the "patternless" representation to be described later.

Finally, we wish to point out that all these human "on-line" optimization methods are inferior in accuracy and efficiency to the above-referenced techniques, which the subjects were in fact trying to simulate. Man's gross computational and data processing ability cannot match well-planned and well-performed calculations in these instances. On the other hand, due to the remarkable human flexibility, man excels in adapting himself to a novel problem by changing an old method or finding a new one, which feature is almost never incorporated in a ready-made computer library routine. A statistical justification of the above statements is beyond the scope of the present work.

The "Analog" Language of Representation.

The subject in this case conceives of the process as being a real analog of some physical or chemical plant, and the control and state variables represent actual physical or chemical quantities.* While the broad behavior of the subjects with this hypothesis was found to be similar to the behavior of those who adopt the mathematical language of representation, the search behavior was more cautious, smaller steps were made and every sudden change in the process output was liable to impose a more conservative attitude on the subject. This fact may be related to the subjects' experience that in real life one often does not have control over all relevant factors and even if he does, the control may be only partial, indirect or inefficient. Some subjects believed that if
there had been a larger number of control variables
the noise level could have been significantly re-
duced. Unexpected, i.e., larger than usually experi-
enced, changes in the response pattern often led to
revisions of plans and "let's start again around
here" statements, as if some mechanical breakdown
could be avoided by a more careful approach.

This hypothesis results in a more detailed and,
on the average, less efficient search behavior than
the mathematical language of representation does.
Experimentalists, people interested in technological
areas are the best specimens of this class, as distinct
from the theoretically inclined representatives of
the previous hypothesis. (See Appendix II.)

The "Counteracting" Language of Representation.
The subject is convinced that the experimenter, di-
rectly or indirectly, is involved in the system, and
the responses he gets as outputs are somehow relat-
ed to his own behavior. This underlying assumption
may prove to hinder the searching activity and re-
sult in a statement:

"Well, whatever I do you can counteract it," or
"Now you have upset my plans again."

The way in which counteracting takes place was
thought to be from either outside, on the console
(although subjects could well see that only input
data, requests for changes in the output frequency
and certain initial loading instructions were typed
in) or from inside, through prearranged tables.

None of our subjects started out with this atti-
tude and neither did they sustain it for long periods
of time. Some, particularly the one who had not
seen a computer before, were more likely to adopt
this viewpoint than others, after a sudden dose of
frustration. Every time the "counteracting" idea
struck a subject, his aspiration level dropped, he
lost his drive and became uninterested. These facts
revealed themselves through the subsequent forcedly
random search, which completely disregarded all
the previous results, and also there were visible
signs of the subject's losing interest.

We cannot of course speak in this instance about
the efficiency of the language of representation, it
is an episode-type event which contributes negative
aspects of insecurity and uninterestedness to the
search behavior.

*One subject, disregarding the fact that there were three
independent variables instead of two, considered the en-
vironment as some terrain and the C values representing
the height of mountains, hills, and valleys. His "hill-
climbing" was a tourist's excursion.

The "Causal Network" Language of Representation.
The subject may build up the image of a casual net-
work of some sort. The control and state variables
are in direct interaction with each other and the
upper limit of complexity of the causal connections
is only determined by the subject's cognitive capa-
bilities. The plans and strategies can become very
complicated. The difficulties may discourage the
subject and lower his level of aspiration. Intermedi-
ate successes would change this situation again. The
net result of the search behavior is similar to the
univariate method mentioned under the mathemati-
cal language of representation. The time-depend-
ence of the background noise is considered to be
due to some mechanical vibrations, and sudden
peaks due to resonance effects.

In general, it can be said that a rather inefficient
and unstable behavior originated from this hypothe-
sis. It occurred rather rarely and could be deduced
from recent educational experience. (See Appendix
II.)

The "Personifying" Language of Representation. A
language of representation, in some respects akin to
the above mechanistic causal networks, can develop
in the subject's trying to personify individual vari-
ables. He describes the "behavior" of the variables
in anthropomorphic terms, such as:

"Well, y2 seems to run away from y1," or
"Now y3 tends to approach y2," or
"You see, y1 is not concerned with x1, it has
a mind of its own", or
"Perhaps y1 is the leader, maybe because it
comes up first and then y2 has to take some
number because y1 took one particular kind and
then y3 comes along and he doesn't have to . . .
he can choose anyone that he wants," etc.

This attitude, however naive it appears, can be
quite common among people with specific educa-
tion backgrounds, such as drama students, or among
people who spend much time in Nature. (These in-
ferences, too, are based on small samples and their
statistical validation is not possible.)

The search is rather haphazard and nonsystematic.
The subject picks some issue of no apparent import-
ance ("I wonder what happens if x1 jumps from 0 to
50 and back, all the time") and sticks to it quite
some time. The experiment seems to him a purpose-
less but enjoyable game void of risk and responsi-
bility.
The "Patternless" Language of Representation. Finally, there can be subjects who are unable to construct any model of the process and believe that the output values, independently of their selecting the levels of the control variables, are "completely random" and do not represent any patternlike phenomenon. This mode of behavior often resolves itself into one of the above languages of representation and also it may happen that, after sufficient disillusionment about one, the subject "gives up" and resorts to this nihilistic viewpoint.

This language of representation, like the "counteracting" type, is only an episode with most subjects. It reduces their drive and interest to an extent that may depend on the individual, his expectations, and the length of time spent with this hypothesis. The above-listed representations have occurred with varying frequency. The subjects occasionally replace one with another, sometimes without trying to explicitly justify the changeover, sometimes after obvious dissatisfaction over its failure. These hypotheses serve as frameworks for the subjects who want to find relevance and meaning in the structure being explored. There was an interaction not only between the representation and the search behavior but also between certain features of the task environment and the adopted hypotheses. We have purposely selected task environments that might induce the subjects to exhibit particular types of associations. For example if the process functions were fairly simple, smooth and not highly periodic, the mathematical language of representation was to be expected. The "analog" representation required also some discontinuities, a small number of steep rises and falls, and moderate interactions between control variables. The "counteracting" hypothesis is more likely to occur if the sum of two or all the three cost functions remains approximately constant, regardless of the changing control variable values. (The subject observes no effect of his attempts.) The "casual network" representation was selected when the state variables were some functions of type (5) noise. (The interaction between control variables is rather obvious.) Similar relations should lead to the "personifying" language of representation. Finally, the "patternless" hypothesis is more often encountered if significant or exclusive noise terms appear in the cost functions.

THE SIMULATION PROGRAM

The aim of the simulation program is, in making use of the characteristic features of the verbal records of the subjects' rules of action, methods, procedures and strategies, to make the computer behave in a way that is essentially similar to the human behavior. This classical Turing's test demands that not only the decisions be the right ones but also the reasons that lead to the decisions be the right ones. With a successful simulation program we can investigate all the possible implications of the theory inexpensively and effortlessly by running many computer experiments in different task environments and with "different" subjects.

Flow charts have been constructed and hand-simulated for every language of representation with the exception of the "personifying" type. The executive routine, the initial search and the mathematical representation have reached the running program stage.

In the following we discuss some aspects of the program only. A more detailed discussion would exceed the scope of this paper.

As mentioned in the Introduction, the program is not a completely deterministic model, it also incorporates stochastic elements. The introduction of the concept of probability into the description of human cognition is considered here a quantitative means of getting around our quasi-ignorance about many minute causal effects. It is, for example, not essential to know which subject selected 8 and which selected 12 trial points during their initial search in the same task environment. It is sufficient to make certain that this number is not correlated in any obvious way with the subject's personality, if we may use such a broad term for the totality of known character descriptives, and that neither does it depend apparently on the outcome of the individual initial trials. We determine the range of this quantity (in this case between 6 and 15) and generate a random number within it for the extent of the simulated initial search.

In general, certain parameters are provided as input for the executive routine. These describe probability distribution functions of which samples are taken to serve as program switches (e.g., type of initial search) or event counters (e.g., extent of initial search).

*Compare the "strategies of decision making" as described in reference 20, or the TOTE units described in reference 21.
In a similar fashion, the initial aspiration level and another reasonable-looking parameter, the level of cautiousness are determined. The respective lower bounds are specified for the executive routine, which then adds small random numbers to these to obtain the actual values.

The adjustment and testing of the aspiration level take place in the following way. The current aspiration level, at first equal to the initial aspiration level, is increased by one every time the program finds a new point with a better “total cost,” i.e., with a value lower than the highest one in the group of best points so far.* Also, the current aspiration level is decreased by 1 after a cycle of 3-7 failures of finding a better point. (Compare this with the readiness of humans to strive for higher goals, as contrasted with the reluctance in regard to lowering ambitions.)

Whenever the absolute value of the difference between the initial and the current aspiration levels exceeds a certain constant, the program quits the first stage of the mathematical language of representation, in which the subject is only concerned with the total cost, and enters the second stage. In this, he tries to establish relations between individual control and state variables. This changeover may, therefore, be due either to having reached a certain level of success (“let’s see what more could be done”) or to some feeling of frustration (“I ought to try some more sophisticated approach now”).

An “abstracting subroutine” makes the program disregard some process outputs between two subsequent input operations at certain times. On other occasions it calculates rough averages of process outputs over small localities of the \( (x) \) space.

The executive routine provides a “public memory,” which the different languages of representation can make use of. It also has monitoring and interrupting power at certain decision junctures. It may happen at one of these interruptions that the program of another representation takes over the control of the hierarchy of elementary information processes.

**DIRECTIONS OF FURTHER STUDIES**

Firstly, there are several possible refinements within the present framework of the project. For example consideration is being given to providing the simulation program with a sort of “learn to learn” feature. In this a record of the relative successes of different representations could influence the hypothesis-selecting mechanism. A much more significant achievement would be if the character of individual task environments could be evaluated and used by the hypothesis-selecting mechanism to find the most efficient representation for the problem at hand. Also, with more experimental data, the probability distributions of the employed parameters would become less tentative.

A possible extension of the present work, which has employed linear and highly regular cost functions, would be to introduce discontinuous, saturated or hysteresis-type cost functions and compare search behaviors in systems with different cost structures while the process itself remains the same.

Another interesting comparison could be made between the decision making processes of two groups of engineering students. The same task environment would be used but one group of students would be told explicitly that the variables of the experiment represent actual, highly critical physical quantities, say \( x_1 \) is temperature, \( x_2 \) is pressure, etc.

A different area of research could be pursued on game playing, game learning, coalition formation, etc. with similar experimental apparatus if certain subsets of control variables are assigned to different players. The participants can be situated in separate rooms, in front of remote consoles.

**CONCLUSIONS**

Computers have been found flexible and efficient tools for providing task environments in psychological experiments. With the development of multi-console time-sharing systems, this endeavor is becoming economically quite feasible on large machines as well. The subjects' total behavior can be reasonably faithfully observed by combining the computer's data processing abilities, for the overt patterns of behavior, with tape-recording the verbalized thinking processes.

The CIP approach has proved to be able to give a causal description of an interesting and rather important facet of human behavior, decision making under uncertainty and risk. It is realized that the particular task environments employed may have had significant effects on the details of the behavior under study, but it is contended that the general as-

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*One must not become confused by the directions of change: the aspiration level *increases* if a better, i.e., *lower*, \( C \) value has been found, and vice versa.

From the collection of the Computer History Museum (www.computerhistory.org)
sumptions and consequences of the theory are independent of the characteristics of different task environments. On the other hand, to force every aspect of a certain type of human behavior into the Procrustean bed of a running program is obviously far-fetched. It is the intention of the present author to generalize this work to cover other phenomena of decision making while remaining as parsimonious as possible.

ACKNOWLEDGMENTS

The author is indebted to Professor Allen Newell for suggesting this research area, and to him and Professor Herbert A. Simon for many stimulating discussions.

REFERENCES

1. The interested reader’s attention is drawn to the following, far from exhaustive, list of papers and books, and to the extensive bibliographies included in them:


APPENDIX I

MATHEMATICAL DESCRIPTION OF THE TASK ENVIRONMENTS USED

In the following, four task environments are specified. The meaning of some symbols is given first.

\[ i = \text{the number of times the subject has named input values;} \]
\[ k = \text{the number of times the subject has received output values;} \]
\[ \delta_{a,b} = \begin{cases} 0 & \text{if } a \neq b \\ 1 & \text{if } a = b \end{cases} \quad \text{Kronecker's delta function;} \]
\[ a=b\pmod{m}: a \text{ is said to be congruent to } b \pmod{m} \text{ if } a-b=km, \text{ where } a, b, k, \text{ and } m \text{ are integers.} \]

We used the shorthand notation \( b \pmod{m} \), representing the remainder of the division \( b/m \);

\[ \Delta x_i = x_i^{(t)} - x_i^{(t-1)}; \] is the difference between the values of control variable \( x_i \) specified currently and on the previous occasion. In all other instances only the current values of variables are considered.

Task Environment 1

\[
y_1 = 80 \sin \frac{x_2}{100} - 60 \sin^3 \frac{x_3}{100} \pi + 10 + 5x_2 + 8i \pmod{21} \\
y_2 = 0.008 x_1^2 - 0.6 x_1 x_2 + 10 x_1 + 60 + 8 \sin k
\]

\[
y_3 = 1 + 10. \delta_{k, 0} \pmod{7}, \\
y_4 = 0.1 x_1^2 - 1.8 x_2 + 30 + 10 x_1 + 5 (|\Delta x_1| + |\Delta x_2| + |\Delta x_3|). \\
\]

Task Environment 2

\[
y_1 = 60 \sin \frac{x_1}{10} \pi + 0.04 x_1^2 - 1.2 x_1 + 70 + \\
\frac{20}{|x_1 - x_2| + 1}
\]

\[
y_2 = 250 - 200 \delta_{x_9, 9} + 20 \cos \frac{i}{5} \pi + 3 k \pmod{16}, \\
y_3 = 200 - 2 (|x_1 - x_2| + |x_2 - x_3| + |x_3 - x_1|).
\]

Task Environment 3

\[
y_1 = 15 \sin \frac{i}{5} \pi + 3 \sin \frac{k}{2} \pi + 60, \\
y_2 = \frac{300}{x_3 + 2} + 4x_3, \\
y_3 = 258 - \frac{300}{x_3 + 2} - 4x_3.
\]

Task Environment 4

\[
y_1 = 150 \sin \frac{x_1}{10} \pi - 0.05 x_1^2 + 6x_1 + 100 + \\
\frac{1}{100} |x_2 - 8|. |x_2 - 27|. |x_2 - 36|, \\
y_2 = \frac{1}{300} |x_1 - 13|. |x_2 - 41|. |x_2 - 24|, \\
y_3 = 430 - y_1 - y_2 + 71 i \pmod{52}.
\]

APPENDIX II

Summary of Experimental Results

The following table contains the summarized results of experiments with 16 subjects. Although this number may appear rather small, the amount of information collected was considerable. The transcript of tape-recorded protocols exceeds 100 typewritten pages.

The Educational Background column of the table refers to the major subject of the person. The last column indicates a score (poor, far, good or excellent) that roughly reflects the quality of the subjects' search and the level of the minimum he was able to achieve by the end of the experiment.
Table 1. Summarized Results of the Experiments.

<table>
<thead>
<tr>
<th>Subject No.</th>
<th>Sex</th>
<th>Educational Background</th>
<th>Task Env. No.</th>
<th>Adopted Language(s) of Representation</th>
<th>Total Number of Trials/Task Env.</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M</td>
<td>Grad. stud. in Eng.</td>
<td>2 and 3</td>
<td>Analog</td>
<td>48 + 33</td>
<td>Good</td>
</tr>
<tr>
<td>2</td>
<td>M</td>
<td>Grad. stud. in math.</td>
<td>1 and 2</td>
<td>Mathematical</td>
<td>59 + 47</td>
<td>Exc.</td>
</tr>
<tr>
<td>3</td>
<td>F</td>
<td>Undergrad. in psych.</td>
<td>1 and 2</td>
<td>Mathematical and counteracting</td>
<td>38 + 36</td>
<td>Fair</td>
</tr>
<tr>
<td>4</td>
<td>M</td>
<td>Undergrad. in drama</td>
<td>4</td>
<td>Patternless</td>
<td>39</td>
<td>Poor</td>
</tr>
<tr>
<td>5</td>
<td>F</td>
<td>Undergrad. in psych.</td>
<td>1</td>
<td>Mathematical and patternless</td>
<td>87</td>
<td>Fair</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>Undergrad. in design</td>
<td>2</td>
<td>Personifying and patternless</td>
<td>48</td>
<td>Poor</td>
</tr>
<tr>
<td>7</td>
<td>F</td>
<td>Undergrad. in bus. &amp; social</td>
<td>3 and 4</td>
<td>Analog</td>
<td>41 + 54</td>
<td>Poor</td>
</tr>
<tr>
<td>8</td>
<td>M</td>
<td>Undergrad. in physics</td>
<td>2</td>
<td>Causal network</td>
<td>45</td>
<td>Good</td>
</tr>
<tr>
<td>9</td>
<td>M</td>
<td>Undergrad. in chem.</td>
<td>2 and 4</td>
<td>Analog and patternless</td>
<td>35 + 31</td>
<td>Fair</td>
</tr>
<tr>
<td>10</td>
<td>F</td>
<td>Undergrad. in drama</td>
<td>3</td>
<td>Personifying</td>
<td>33</td>
<td>Poor</td>
</tr>
<tr>
<td>11</td>
<td>M</td>
<td>Grad. stud. in math.</td>
<td>1 and 4</td>
<td>Mathematical and patternless</td>
<td>52 + 49</td>
<td>Fair</td>
</tr>
<tr>
<td>12</td>
<td>F</td>
<td>Undergrad. in nat. sc.</td>
<td>2</td>
<td>Analog and patternless</td>
<td>40</td>
<td>Fair</td>
</tr>
<tr>
<td>13</td>
<td>M</td>
<td>Undergrad. in psych.</td>
<td>1</td>
<td>Mathematical</td>
<td>56</td>
<td>Fair</td>
</tr>
<tr>
<td>14</td>
<td>F</td>
<td>Undergrad. in nat. sc.</td>
<td>3</td>
<td>Causal network and counteracting</td>
<td>68</td>
<td>Poor</td>
</tr>
<tr>
<td>15</td>
<td>F</td>
<td>Undergrad. in psych.</td>
<td>4</td>
<td>Patternless</td>
<td>37</td>
<td>Poor</td>
</tr>
<tr>
<td>16</td>
<td>M</td>
<td>Undergrad. in math.</td>
<td>4</td>
<td>Mathematical</td>
<td>64</td>
<td>Exc.</td>
</tr>
</tbody>
</table>

Appendix III

COMPARISON BETWEEN EXCERPTS FROM A REPRESENTATIVE PROTOCOL AND COMPUTER PERFORMANCE

In both instances, in the experiment and in its computer simulation, Task Environment 1 was used with the simplest cost function, in which \( c_i = y_i \). Editorial remarks are put in square brackets.

The subject's initial search here could be classified as systematic with slight random components. He adopted the Mathematical Language of Representation without any hesitation. In Stage (A) he followed a scheme fairly thoroughly but was not satisfied with the rate of improvement and entered Stage (B). In this, he employed the so-called univariate method which consists of changing only one control variable at a time. The optimization takes place in terms of single control variables and at the end an attempt is made to specify a global optimum.

* * *

\( E \): Can we start now?

\( S \): O.K., let's try around zero, say, 2, 3, and 4. [i.e., \( x_1 = 2, x_2 = 3, \) and \( x_3 = 4 \); the first output followed: \( c_1 = 42, c_2 = 77, c_3 = 90, \Sigma c = 209 \).] Well, let us try another type... [interrupted by a new output: \( c_1 = 42, c_2 = 83, c_3 = 90, \Sigma c = 215 \) ... hmm, it's increased ... O.K., let them be 45, 5, 4. [i.e., \( x_1 = 45, x_2 = 5, x_3 = 4 \); output \( c_1 = 60, c_2 = 31, c_3 = 698, \Sigma c = 789 \).]

In describing the computer's actions, instead of articulated English sentences, we can only refer to brief statements, such as EXTRAPOLATION, INTER-
POLATION, ASPIRATION LEVEL NOW:, COUNTER NOW:, NEW POINT:, etc. These had been planted in the program to indicate at appropriate times what action the machine is taking. The following excerpts from the trace of the program [P] are approximately equivalent to the segments of human behavior at the left, almost paragraph by paragraph.

* * *

INITIAL SEARCH TYPE 1.
NEW POINT: X1= 3,X2=47,X3= 49.
OUTPUT: C1=273,C2=84,C3=196,
SUM=553.

OUTPUT: C1=273,C2=90,C3=196,
SUM=559.

[The noise appears here.] 
NEW POINT: X1= 4,X2= 2,X3=0.
OUTPUT: C1= 36,C2= 98,C3=541,
SUM=675.

NEW POINT: X1= 5,X2= 48,X3= 3.
OUTPUT: C1 =260, C2= 102, C3 =474,
SUM=836.

Whoops, it’s up ... [interrupted by a new output:
C1 =60, c2=29, c3=698, Σc=787]. Now, let them be
4, 48, 48. [Output: c1=273, c2=90, c3=754, Σc=
1117].

Oh, that’s bad. 0, 50, 0. [Output: c1=271, c2=52,
c3=460, Σc=783.]

Still no good, let’s mix them further ... [interrupted by output: c1=271, c2=50, c3=460, Σc=781]
... Well, x₂=45, x₃=48, x₃=4. [Output: c1=278,
c₂=27, c₃=879, Σc=1184.] This is the worst so far.

[Output: c₁=173, c₂=83, c₃=617, Σc=873.]

E: How did you pick these values? Can you explain it now?

S: Well, I thought I would first try all these corners ... a sort of local search at the two ends of every x range, and also in its middle. This might give me some idea of how that sum behaves ... It could, of course, go up and down in between ... but that may not be important. I hope these functions [the hypothesis is formed!] do not oscillate too much ... 

OUTPUT: C1=260, C2=104, C3=474,
SUM=838.

NEW POINT: X1= 24, X2= 27, X3= 22.

OUTPUT: C1=191, C2= 64, C3=589,
SUM=844.

[P names nine points with coordinates either at the two ends of the ranges, 0 ≤ x ≤ 5 and 45 ≤ x ≤ 50, or in the middle, 20 ≤ x ≤ 30.]

THE MATHEMATICAL LANGUAGE OF REPRESENTATION IS ACCEPTED.
INITIAL ASPIRATION LEVEL: 8.

S: Now I have ... how many ... four, five reasonable points. The sums here are no worse than say, 800. Let us be careful now ... I want to give you a good x₁ ... It’s better if I don’t care about these continuous outputs now ... Well, when x₁ was around zero, the sum was about 210; when it was 28, we had almost 600. So, how about ... how about, say, if x₁ equals 10 ... that might hit the minimum ... This kind of thing can give us, shall we say, 15 for x₂, and ... well ... I’m doing the same for x₃ ... O.K., let it be 20. [Output: c₁=134, c₂=108, c₃=736, Σc=778.] It didn’t do much good ... O.K., let us include this point as well ...

SELECTED SUBSET OF POINTS:
X1= 3, X2=47, X3= 49, SUM=553;
X1= 4, X2= 2, X3= 0, SUM=675;
X1= 4, X2= 3, X3=50, SUM=680;
X1=46, X2= 4, X3= 1, SUM=779.
HUMAN DECISION MAKING UNDER UNCERTAINTY AND RISK

P selects four points out of the nine “non-noisy” ones, with the lowest total cost values.
INTERPOLATION: X1 = 21;
\[=3+(46-3) \frac{553}{657+779}\]
INTERPOLATION: X2 = 22;
\[=2+(47-2) \frac{675}{657+553}\]
INTERPOLATION: X3 = 25;
\[=0+(50-0) \frac{675}{657+680}\]
NEW POINT: X1 = 21, X2 = 22, X3 = 25.
OUTPUT: C1 = 130, C2 = 97, C3 = 563,
SUM = 780.
OUTPUT: C1 = 130, C2 = 103, C3 = 563,
SUM = 786.
COUNTER NOW: 1.
P fails to hit upon a sum better than at least 779,
by interpolation. It is counting the number of failures.

SELECTED SUBSET OF POINTS:
X1 = 3, X2 = 47, X3 = 49, SUM = 553;
X1 = 4, X2 = 2, X3 = 0, SUM = 675;
X1 = 4, X2 = 3, X3 = 50, SUM = 680;
X1 = 46, X2 = 4, X3 = 1, SUM = 779;
X1 = 21, X2 = 22, X3 = 25, SUM = 780;
X1 = 5, X2 = 48, X3 = 5, SUM = 804.
P selects 6 points out of the 10 “non-noisy” ones
so far, with the lowest total cost values.

S: I just can’t get a better point ... This is silly.
Why don’t I move off? Suppose, we just try 35, 40,
45 for a change.

S: [After hitting a point with C = 581]. Here we are,
that’s reasonable ... I see it now ... This thing
seems to move downwards, toward the smaller x’s.
Let us choose ... well, x1 was all right, let it stay at
8, ..., x3 has to be down ... down ... say, 15 and
x3 stays in the middle, 25.

E: What is wrong?
S: I just don’t get that sum any better. I somehow
feel, I have to do something about c’s ... I mean,
about c1, c2 and c3 separately. Maybe I should mini-
mize c1 only, at first. Let c2 and c3 be whatever they
are, and then come back to them ... Yes, that’s what
I am going to do. Minimize them, one by one, and
then get them together again. Where was c1 the best?
COUNTER NOW: 5.
RANDOM BREAK-OUT.
ASPIRATION LEVEL NOW: 7.
NEW POINT: X1 = 17, X2 = 38, X3 = 9.
[After five failures the aspiration level drops and
the next point is specified at random.]

S: Well, c1 depends on x2 and x3, and a little bit,
maybe a little bit, on x1. And it also varies with time
... Yes, low values of x2 are all right ...

S: You see, this was easy, to minimize c2 ... I
now know what it is like ...

S: That c3 is tough, I can’t see much reason behind
it. Somehow, when I don’t move with the x’s, it’s
smaller, and when I change them a little, it jumps up . . . It is a funny variable . . .

POINTS WITH BEST C1:
X1 = 4, X2 = 2, X3 = 0, C1 = 36;
X1 = 18, X2 = 5, X3 = 2, C1 = 52;
X1 = 28, X2 = 4, X3 = 6, C1 = 57.

[A further search produces these three best points with regard to C1]

POINTS WITH BEST C2:
X1 = 35, X2 = 11, X3 = 47, C2 = 39;
X1 = 45, X2 = 38, X3 = 3, C2 = 47;

POINTS WITH BEST C3:
X1 = 3, X2 = 47, X3 = 49, C3 = 196;
X1 = 27, X2 = 13, X3 = 5, C3 = 343;
X1 = 6, X2 = 7, X3 = 21, C3 = 352;
X1 = 11, X2 = 15, X3 = 34, C3 = 355;
X1 = 3, X2 = 4, X3 = 18, C3 = 374.

S: If I take, say, 36 for X1, 5 for X2 and . . . and a small value, say, 2 for X3, I should get just about the minimum . . . I don’t think I can do any better . . . not when this noise is on all the time. | Output: C1 = 48, C2 = 21, C3 = 408, C4 = 477. End. |

THE BEST POINT SO FAR:
X1 = 28, X2 = 2, X3 = 47, SUM = 463.

The following points may be worth mentioning with regard to the comparison between the above two records:

The simulation of both the results of and the reasoning behind the subject’s decision making is fairly faithful, although the trial points are, of course, not identical. The only serious shortcoming of the model can be seen at Stage (B), when it does not notice the effect of large step sizes. Consequently, the program’s best points with regard to C3 just, so to speak, happen to be the best. The first point on the list (X1 = 3, X2 = 47, X3 = 49, C3 = 196) is “too good” and its weight causes the final selection mechanism to choose a “global optimum” with a much too large X3. (The X3 = 47 value of the first point on the best—C3—list was also guilty in this decision.)

The quality of the computer search for minimum was also very similar to the human one. The machine obtained a minimum of 463 after 68 trials, as contrasted with the subject’s minimum of 477 after 59 trials.