The Measurement of Social Change

RICHARD L. MEIER

Is it possible to build a synoptic instrument, similar to a telescope or a radar network, for viewing one's own society? How may we interpret the myriads of social activities that are presently undertaken? Preliminary explorations suggest that we need sensing techniques, or transducers, that pick up changes going on inside the society. External indicators, like air photos, are too superficial. We are faced with a problem of discovering what operating characteristics a deep-probing instrument should have so that it may be as practical and useful as possible.

Economists judge change in society by modifications in the makeup of the gross national product and the level of expenditures, political scientists can analyze elections and polls, but sociologists and anthropologists have no cumulative sets of accounts or aggregate indexes. They have had hopes, however, similar to those expressed by Lazarsfeld [3]:

Our economic statistics are today quite well advanced. We know how much pig iron is produced and how much meat is exported every year. But we still have very little bookkeeping in cultural matters. The content of mass media of communication is an important and readily available source of social data, and it will not be surprising if this analysis becomes a regular part of our statistical services in the not too distant future.

These statistics have not yet come into being because the labor cost was high, the time lags were great, and the system description was incomplete, so it has been impossible to state how one set of measurements related to another.

Let us take a brief, searching look at the social system. Society is maintained and changed by the behavior of its members. Intuitively one feels that the basic unit of behavior is the act, but acts are not as easily counted and differentiated as particles, molecules, or organisms. Satisfactory data can only be obtained when actors are forced to confine their behavior within certain preset specifications or codes, which may be called languages, currencies, habits, or "standard operating procedures." This behavior must be observed in public spheres, since the objective, detached observer is missing in private affairs. The latter will require altogether different instruments and techniques for data accumulation, and will not be taken up here.

By far the most promising attack upon the problems of measurability is offered by lumping together small sets of acts into transactions. A social transaction involves, among other things, the emission of a message together with evidence of its receipt—apparent to an observer, but also, through one or another form of feedback, to the agency responsible for emission.

At any given moment, the population of the society can be divided into senders, receivers, and nonparticipants, much as the economist divides his population into producers and consumers, and each participant must play both roles. The message normally contains some information that is novel to the receiver, more that is redundant, and some symbols that are quite unintelligible. Some messages are not communicated directly to receivers, but are stored in libraries, files, and artifacts where they become a resource embedded in the social environment. Uncoded information may be gleaned from the environment through systematic observation. Scientists, weather observers, diagnosticians, and other professionals have been trained to reduce these phenomena to coded, communicable form (Fig. 1). The information flows should be sampled where the wavy lines occur.

The term "information" at this point has been used in its intuitive sense. At a later stage, it will be shown that the demand for information storage (used now in its technical sense) in our instrument corresponds crudely with the volume of these flows in society—about as well as national income figures represent the combined satisfactions of consumers. The greatest difficulty in the design of our instrument is the conversion of all of the codes for human communication, oral, written, graphic, gestural, musical, etc., into a single code.

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which is convenient for machine handling. We may have to incorporate human operators, whose skills resemble those of cataloguers in a library, for the more difficult features of translation. Fortunately, as we shall see later, the bulk of the information flow in modern society is in the form of printed language, which seems amenable to automatic sensing, coding, and abstracting (Luhn [4]).

Given the complexities in social communications, how would a representative and comprehensive overview of the social transactions be obtained for our instrument? The mass media—television, radio, magazines, newspapers, books, records, catalogues, direct mail advertising, etc.—could be recorded at the source. Schools, conferences, committee meetings, shop talk, live performances, etc., would have to be random-sampled. But on what basis?

Here we are forced to refer to a fundamental property of modern society. A sender can have many simultaneous receivers, but any given receiver usually accepts messages from but one sender at a given moment; on rare occasions he may pay attention to two or three, but no more. The decision as to the completion of a social transaction depends upon the receiver. He pays for the message by spending time taking it in. Human time is a moderately scarce commodity, and cannot be wasted indefinitely. People tend to switch dials and scan the newspapers and magazines until they find messages that are interesting to themselves. Message types that gain few receivers tend to be dropped by senders in favor of those which get more attention. Therefore, the social value of broadcast messages may be determined, as a first approximation, by the amount of time people devote to them.

Thus a comprehensive time budget of the members of the society—how they allocate time to receiving messages of various kinds, and time to other matters that do not involve social communications—provides a simple, additive criterion of value. We could attach the probable number of receivers, with some estimates of the time and place of reception, to the records of the messages themselves that are held in storage for our instrument.

The formulation of a society-wide time budget has already been explored by Meier [5]. A quantitative description of time-use has applications in public affairs independent of its employment in our system, and the techniques required for making economical measurements already exist.

Economists have found that macro-analysis is greatly assisted by subdividing the economy into such sectors as agriculture, manufacturing, households, etc. The rules for simplifying the accounting may be different in each sector. The choice of sectors in social analysis will depend upon the kinds of reinforcement provided by other approaches to social measurement, such as public opinion evaluation, the Census, and historical analysis. A first guess regarding sectors is provided in Table 1.

Another feature of our instrument must be introduced. If it is to be economically constructed, it should be decentralized. The headquarters would contain communications which are subject to national distribution, plus some measurements of exports and imports over continental boundaries, while branches would exist in every metropolitan area (Fig. 2).

In the course of proposing a design for this apparatus, we have piled feature upon feature so that it has by now become quite elaborate. It is expected to intercept and store a huge volume of messages, but this is made feasible by eliminating most of the redundancy in social communications, and reducing all the messages to a common code. The instrument attaches weights to these messages according to the number of persons and the amount of time spent receiving them; it indicates the times and places the message is received, and it must store all of this in a permanent record which can be scanned quickly and automatically. Fortunately it need not get every message that is received, but may start modestly by sampling at, say, a one per million rate. As the instrument is refined, and the representation of social change that is required must be finer-grained, the sampling rate may be advanced.

We are now ready to discuss who would use such an instrument and for what purposes. Planners and administrators who must make decisions for the public regarding parks, playgrounds, schools, traffic patterns, and various social services should be able to develop criteria for deciding from studies of trends in social communications and from comparisons with other sources of social data. Advertisers may be expected to develop their craft on the basis of the more detailed measurements of response they would be able to obtain. Politicians should be able to sense better the distribution of sentiment on various issues. Educators may assess the impact of special programs. Changing tastes, the appearance of new patterns of social interaction, and the passage of fads should all be registered as factual data—"how much," "where," and "when." The natural emphasis is upon local public affairs, mass entertainment, and the functioning of work, school, and commerce, because these matters make up the bulk of our communications activity.

### Table 1

**Communications-Oriented Allocation of Time in Public Activities**

<table>
<thead>
<tr>
<th>Work</th>
<th>School</th>
<th>Radio and TV</th>
<th>Shopping</th>
<th>Travel</th>
<th>Reading</th>
<th>Meetings and Parties</th>
<th>Dining and Drinking</th>
<th>Play</th>
<th>Ritual</th>
<th>Personal Services</th>
<th>Miscellaneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factory</td>
<td>Office</td>
<td>Construction and Mining</td>
<td>Agriculture and Forestry</td>
<td>Housework</td>
<td>Maintenance of Property</td>
<td>Services</td>
<td>Maintenance of Property</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>Services</td>
<td>Maintenance of Property</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Possible Subcategories under Work**

- Housework
- Maintenance of Property
- Services
- Miscellaneous

From the collection of the Computer History Museum (www.computerhistory.org)
Meier: The Measurement of Social Change

Fig. 2—Organization of social communications as imposed by the location of various activities.

A skilled operator would ask his questions in terms of key words or phrases appearing in the content with a frequency of $10^{-6}$ to $10^{-8}$. They serve as "tracers" of message content as it is spread through the population. Maps and time series can be prepared which show their buildup and decline. More detailed information about the changing attitudes of people may be obtained by reconstructing the contexts within which the key words appeared.

The severest criticism to be made of a representative record of social communications is that the content of the messages tends to be superficial. In many, if not most, social transactions people disguise their true feelings about a subject. An investigator may nevertheless make many nontrivial observations, and can probe more deeply, if he desires, by using the "trial balloon" technique (Fig. 3). An event, closely relevant to the subject of interest, is purposely created—it may be an announcement, an incident, or a rumor. The subsequent wave of "talk" that is stirred up may then be analyzed. The effect that is triggered off provides a good indication of the sensitivity of the public to that issue at that time.

These and other small-scale tactical uses in government and commerce should grow rapidly to the point where large installations may be justified which allow hundreds of simultaneous operators.

The strategic uses of an instrument of this sort are still more interesting. The accumulation of socio-cultural "wealth," for example, may be estimated in a manner analogous to that developed by economists, and the flows of information through society may also be estimated. A very brief outline of the steps involved, and the kinds of conclusions to be obtained, will be presented.

Fig. 3—The "trial balloon" stimulus as revealed by content analysis. Assume the stimulus contains concepts whose treatment in communications uses terms A, B, and C with high probability.

What is the total of all nonredundant information that is transmitted in society for a year? The limitation upon flow is the capacity of the receiver to understand the messages to which he exposed himself. A receiver has a limited repertory of terms. Reasonably good statistics exist only for English vocabulary.

The respective terms that are used in messages can be mapped according to their probability of occurrence in social communications, as in Fig. 4. The abscissa is some arbitrarily defined categorization of meanings, similar to the Dewey decimal system. When this same map is put onto polar coordinates, we can show stages in the development of a receiver as in Fig. 5. The protuberances on the periphery are associated with the specialties engaged in by the person. The map of transition probabilities between terms would have the same appearance.

There is a standing rule in society that a sender should have greater knowledge about the subject of the message than the receiver, if information is to be transmitted. Thus, on the average, the senders are more informed and more expert than the receiver, as shown in Fig. 6. Continued communication would cause the receiver’s repertory of terms to grow in the direction of the sender’s. He would learn something about the subject. Senders must choose their terms so that they lie on the periphery of the receiver’s map, if they are to save time and maintain interest.

We are now in a position to estimate the amount of information flowing that is potentially useful to receivers. Let us define a restricted number of classes of receivers, say about a hundred, each representing a different segment of society, ranging from illiterates to various kinds of professionals, but exhaustive of the...
Categories of social communications, spaced according to the allocation of the volume of transmitted terms to each of them respectively.

In American culture A might include terms used in newspapers, magazines and popular books, B those used in conversation, radio, and television, C those employed in business transactions, etc. Categories or contexts (A, B, C, ... K) are assigned by convention as before.

The levels may be obtained through vocabulary tests, completion tests, etc. They are excellent indices of intellectual achievement in the respective directions.

Fig. 4—Typical properties of a receiver of social communications.

Fig. 5—Typical development of the vocabulary map in an individual. The powers of ten shown are levels for the frequencies with which terms appear. If the Zipf distribution (rank order times frequency is a constant) holds, each shell contains the indicated number of terms. Categories or contexts (A, B, C, ... K) are assigned by convention as before.

Fig. 6—Relationships between repertoires in social communications. The receiver expands his map in the indicated direction as a consequence of communication. Simplified maps showing how the sender chooses terms which lie on the boundary of the vocabulary that is shared, if he wishes to optimize the transfer of information.

Fig. 6 (a) and (b)

TABLE II

<table>
<thead>
<tr>
<th>Mode of Reception</th>
<th>Time Allocated hours/year</th>
<th>Estimated Receiving Rate bits/minute</th>
<th>Estimated Flow bits/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>4X10^4</td>
<td>1500</td>
<td>36X10^9</td>
</tr>
<tr>
<td>Television</td>
<td>3X10^4</td>
<td>500</td>
<td>9X10^9</td>
</tr>
<tr>
<td>Lecture and Discussion</td>
<td>4X10^4</td>
<td>200</td>
<td>5X10^9</td>
</tr>
<tr>
<td>Observation of Environment</td>
<td>3X10^4</td>
<td>100</td>
<td>2X10^8</td>
</tr>
<tr>
<td>Radio</td>
<td>1.5X10^4</td>
<td>300</td>
<td>3X10^8</td>
</tr>
<tr>
<td>Films</td>
<td>1X10^4</td>
<td>800</td>
<td>8X10^8</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>5X10^3</td>
<td>100</td>
<td>3X10^9</td>
</tr>
</tbody>
</table>

per capita average ~10^8 bits/year

* Judged in terms of the probable repertoires of receivers, not in the accepted sense used in information theory. Possibly a new term should be coined for information distributed over a population.
Comparisons between the poorest and richest metropolitan areas can also be exceedingly suggestive (Table III). Observers now agree that socio-cultural growth parallels economic growth but the introduction of measurements suggests that social communications must either precede economic growth or grow more rapidly than income. Apparently an expansion of socio-cultural activity is a necessary but not sufficient precursor of economic development.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>Income and Information Flow Extremes in Urban Society</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>San Francisco</td>
</tr>
<tr>
<td>Income</td>
<td>$3000 capita/year</td>
</tr>
<tr>
<td>Non-redundant* information receipt</td>
<td>~10^8 bits/year</td>
</tr>
<tr>
<td>Addis Ababa or Jakarta</td>
<td>$150 capita/year</td>
</tr>
<tr>
<td></td>
<td>~10^8 bits/year</td>
</tr>
</tbody>
</table>

* Again in terms of probable repertoires of receivers. This assumes that 70–80 per cent of residents in the poorer cities are illiterate. The heavy volume of information transmitted by reading is highly significant. A society like our own which is increasingly white collar reads more at work and at home. There are limits to human ability to receive information, however, which are believed to be in the neighborhood of 10^9 bits per capita per annum for a population with the present distribution of mental capacities. At the present estimated rate of gain, this theoretical saturation level is likely to be reached within two generations. The prospect is startling enough to cause us to investigate more closely the stresses associated with communication saturation in human organizations.

My feeling is that our instrument is already technically feasible. Simple calculations show that sampling social communications at a rate of ten parts per million presents storage requirements within range of existing equipment, but the desired degree of access remains unclear. Message collection in the field is not a problem, but the programming for storage and the cataloguing of nonverbal materials has been inadequately developed. Much experimental work and formal analysis will be required before a truly comprehensive cross section of social change can be achieved.

REFERENCES

Simulation of Sampled-Data Systems Using Analog-to-Digital Converters
MICHAEL S. SHUMATE†

INTRODUCTION
UNTIL recently, systems simulation problems could generally be split into two classes: problems requiring only an analog computer for solution and problems requiring only a digital computer for solution. Any general problem has a number of characteristics which adapt it to either one or the other method of solution.

A systems problem is most readily adapted to analog computer simulation when it requires a relatively short solution time on the computer and a relatively inaccurate solution is acceptable; has relatively "high" frequencies; and has nonlinearities such as, for example, saturation, deadzone, or hysteresis. To be adapted to digital computer simulation, a systems problem usually possesses relatively low frequencies and requires a long solution time; can be adapted to an iterative form of simulation without introduction of an instability (this is usually implied by a lack of nonlinearities such as those mentioned above); and has a range of variable which exceeds that possessed by an analog computer solution.

Certain problems involving combinations of both groups of properties may often be split into two separate problems, one involving high-frequency nonlinear effects, and one involving low-frequency effects.

An example of such a problem is the simulation of the flight of a liquid-propelled ballistic missile. The missile's
reactions to external and internal forces can be simulated on an analog computer, and the motion of the missile along a trajectory can be simulated on a digital computer.

However, recent developments in control systems, particularly systems using sampled or discrete data, have generated a third class of simulation problems; this class of problem usually involves both sampled and continuous information, has nonlinearities and no general dividing line between high and low frequencies, and usually has a range of variable which exceeds that possessed by an analog computer. An example of such a problem is the simulation of an inertially guided missile. The guidance and control systems must have rapid response characteristics, and the system may involve a special-purpose digital computer, whose program makes it act like a multimode, adaptive controller.

**Simulation of Sampled Data Systems**

Large analog simulation facilities are thus faced with the problem of obtaining reasonable representations of such systems, and must thus have the capability of at least sampling continuous information, and hopefully be able to perform some intelligent and useful operations on such information.

Several straightforward methods of performing sampled-data simulation are presently available. Sampling may be accomplished by a so-called hold-amplifier [see Fig. 1(a)] or a passive hold circuit [see Fig. 1(b)]. These circuits are not easily adapted to simulation of digital computer operations, because of difficulty involved in holding past values of sampled information. This difficulty is partially eliminated by using transfluxors as hold devices.

The most general method for simulation of sampled-data systems is to use analog-to-digital converters to connect an analog computer to a general-purpose digital computer (see Fig. 2). In such an installation, one or more analog-to-digital converters are used to sample continuous information and present it to a digital computer. Several more analog-to-digital converters are used open loop as digital-to-analog converters to present and hold digital information to the analog computer. Two major disadvantages of this system are:

1) The difficulty in starting the two computers simultaneously, and
2) The difficulty in scheduling operation time on the digital computer, since in most installations digital computing time is a premium quantity.

The first of these difficulties may be remedied by equipping the digital computer with an "interrupt" feature, and the analog computer with a "start" control. The interrupt feature allows the digital computer to break its program when a pulse (perhaps a sample pulse) is received, and jumps to a special subroutine used for external communication purposes. The start control allows the analog computer hold relays to actuate simultaneously with the same pulse that first interrupted the digital computer.

The second difficulty is easily overcome if one is willing to pay for the digital computer time. It seems a little ridiculous, however, to use a large general-purpose digital computer if the digital program involved is relatively simple. For example, in order to use this system to simulate a sample-and-hold, the digital computer must be programmed to perform a transfer of a number from an analog-to-digital converter to a digital-to-analog converter, and would thus sit idle during most of a sample period.

It therefore becomes evident that such a large, general-purpose sampled data simulator should only be used for complex, sophisticated problems, and some auxiliary equipment should be developed to simulate simpler sampling operations.

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The construction of an auxiliary sampled-data simulator is, of course, dictated by equipment present in a simulation facility. The facility currently available to the author consists of a 300 amplifier Electronic Associated Analog Computer, an EPSCO Addaverter conversion system, and a Remington-Rand Univac Scientific Digital Computer, Model 1103A.

The approach used in the construction of an auxiliary sampled data simulator was based on the fact that the facility currently possessed equipment which would both sample analog voltages and hold analog voltages. This equipment, the Addaverter (see description below), would, of course, be in use part of the time as a communication link to the 1103A, but its time schedule was sufficiently open to warrant thinking of using it for other purposes.

The Addaverter is used to simulate a group of parallel sample-and-hold channels. A sample-and-hold is obtained by effectively placing an analog-to-digital converter in series with a digital-to-analog converter. The analog-to-digital converter is caused to sample its input voltage, the resulting digital number is then transferred to the digital-to-analog converter, and converted back into a voltage, which remains unchanged until the complete cycle is repeated again. Note that delaying of the conversion of the number back into a voltage is equivalent to delaying the sampled values of the input voltage and is useful for simulation of transportation lag, etc.

In order to obtain a better description of the implementation of the above concept into a complete sampled data simulator, some description of the Addaverter is necessary.

The Addaverter consists of 30 analog-to-digital converters (hereafter abbreviated ADC), 15 used as such, and the other 15 used open loop, as digital-to-analog converters (abbreviated DAC).

All 15 ADC's sample the voltages present at their inputs simultaneously, under the control of a central sample controller, which is triggered by a single pulse, called a sample pulse; the resulting digital numbers remain intact in the memory of each ADC until the next sample pulse occurs.

The DAC's operate individually, each converting the digital number previously stored in its memory to an analog voltage when it receives a pulse, called a present pulse. The voltage at a DAC's output remains unchanged until a new number has been read into its memory and a present pulse has been received. A single present pulse input and 15 pulse inhibit inputs are used, instead of 15 pulse inputs.

In order to transfer the digital numbers from the ADC memories to the DAC memories without using the digital computer, an additional piece of equipment, normally used for Addaverter maintenance purposes, is employed. One particular operational mode of this equipment, when triggered by a single pulse, transfers the numbers stored in the ADC memories into the DAC memories in a sequential fashion: the content of the first ADC memory is transferred into an auxiliary register, and then into the memory of the first DAC; the process is then repeated for each of the other 14 ADC-DAC channels. Provision is made to prevent the number transfer for any individual channel by energizing an inhibit gate associated with that channel.

These three units, the 15 ADC's, the 15 ADC's used as DAC's, and the number transfer equipment, comprise the basic sampled-data simulator. In order to obtain satisfactory operation, it is necessary to supply a burst of three pulses each time it is desired for the simulator to sample. Another piece of equipment was constructed to supply the pulse burst necessary to control the simulator. This equipment consists of necessary control logic for the simulator, and a large quantity of pulse and dc logic with a prepatch capability, to permit flexible operation of the entire system. A source of timed pulses is available, to use as system clock; a “start” system is available, to synchronize starting of analog simulations with the clock pulses.

An ADC-DAC channel may thus be used to sample-and-hold an analog voltage with either of two transfer functions:

\[
H_1(s) = \frac{1}{s} (1 - e^{-\tau T}), \quad \text{and} \quad (1)
\]

\[
H_2(s) = \frac{1}{s} (1 - e^{-\tau T}) e^{-\tau T} \quad \tau \leq T \quad (2)
\]

where \(T\) is the sampling period, and \(\tau\) a delay time.

No problems are incurred if it is desired to have all 15 ADC-DAC channels operating in the same mode. However, it is sometimes desired to simulate multirate sampled-data systems, or monorate systems that have several samplers each with a different time delay. No difficulties would arise if each ADC-DAC channel operated independently of every other channel. This is not the case, however, since all channels sample simultaneously.

The simulator control equipment was so designed to permit simulation of up to four different sample-and-hold operations; however, their operating frequencies must be restricted to integral multiples of some frequency. A block diagram of the simulator control is shown in Fig. 3. The simulator control has a set of sample pulse inputs, a set of present pulse inputs, and a set of “enable” outputs, all located on the prepatch panel. The inhibit inputs for the present pulse (DAC...
inhibit), and inputs for the transfer inhibit are wired in parallel and brought up to the prepatch panel. The four “enable” outputs, one for each operating group, must be patched to the inhibit inputs of the channels, as predetermined by the programmer. If a particular channel is to operate according to the group 1 mode, its inhibit input must be patched to the group 1 enable. Several channels may operate from the same group enable.

A particular group’s operating mode is completely determined by what pulses are fed to its sample input and its present input. For example, to operate a group as a sample-and-hold [see Fig. 4(a)], a clock pulse must be fed to the group sample input each time it is desired for a sample to occur. A present pulse must then be fed to the group present input 815 μsec after each sample pulse. The sample input pulse causes all group enable outputs to be reset, then the particular group enable (whose input was pulsed) is set, and the ADC’s are caused to sample (all 15 sample) 180 μsec later, the number transfer is initiated. (Only those channels associated with the operating group have numbers transferred.) The present input pulse, which must be delayed at least 815 μsec to allow the sample and number transfer to be completed, then causes the DAC’s connected to the operating group to present the numbers transferred as voltages.

Note that the sample-and-hold is actually a sample-and-hold plus a short delay; this delay is not long enough to introduce unwanted effects in the majority of problems simulated. If it is desired to introduce more delay, it is only necessary to increase the delay of the group present input pulse. The maximum delay possible for a group present input pulse is the sampling period for the particular group.

To operate two groups at different, related frequencies, the group running at the higher frequency is operated as described above, but the clock pulses for the lower frequency group must be obtained by dividing the higher frequency clock pulses by some integral number. [See Fig. 4(b).] The start control input is shown in Fig. 4(a) and 4(b).

A great many other modes of operation are possible, but will not be presented here because of space restrictions.

Simulation of Simple Digital Computer Operations

Simulation of sampled-data systems which incorporate a digital computer may be accomplished by using sample-and-hold channels to simulate the digital computer’s transfer function (provided the transfer function is not too complicated). Simple digital filters, second- or third-order difference equations, etc., may be simulated.

If a group of ADC-DAC channels are wired in series [the output of the first wired to the input of the second (see Fig. 5, with the control logic being pulsed by logic circuitry given in Fig. 4(a) of the example logic diagrams, the variable delay set to 815 μsec, and all channel inhibit inputs wired to the operating group enable

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Footnote: Logically, if a number is transferred into a DAC, it is done so with the intention of presenting it.

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output], then it can be easily shown that successive channel outputs are each delayed from the previous channel output by one sample period.

$E_n^*$ in this treatise will be used to denote the \textit{sampled and held value} of $E$; the subscript $n$ refers to the $n$th sample pulse.

Suppose $E$ is zero, and has been for a considerable length of time. Then $E_n^*$, $E_{n-1}^*$, etc., will all be zero. Now suppose that during the interval between the $n$th and the $n$th + 1 samples, $E$ changes to some nonzero variable value. Then, after the present pulse associated with the $n$th + 1 sample, channel 1’s output will change to $E_{n+1}^*$. However, during the sampling associated with the $n$th + 1 sample, channel 1’s output was still zero, hence the output of channel 2 will remain at zero after the $n$th + 1 present pulse.

After time $n+2$, the output of channel 2 will have the value $E_{n+1}^*$, since that value was still at the output of channel 1 when the $n$th + 2 sampling occurred. The output of channel 3, $E_n^*$, will still be zero. (This operation is a consequence of the fact that each sample-and-hold channel has a slight transportation lag.)

Therefore, a series string of sample-and-hold channels will act like a delay line which propagates values of $E$ that have been sampled and held. Further consideration will show that this is equivalent to the way a digital computer would remember past values of a variable.

Thus

\begin{align*}
\text{Output of channel 1} &= E_n^* \\
\text{Output of channel 2} &= E_n^* e^{-\alpha T} \\
\text{Output of channel 3} &= E_n^* e^{-2\alpha T}.
\end{align*}

Using $z$ transform notation ($z = e^{\alpha T}$; see Ragazzini and Franklin)

\begin{align*}
\text{Output of channel 1} &= E_n^* \\
\text{Output of channel 2} &= E_n^* \frac{1}{z} \\
\text{Output of channel 3} &= E_n^* \frac{1}{z^2}.
\end{align*}

The first channel has the transfer function

\begin{equation}
G_1 = \frac{1}{z} (1 - e^{-\alpha T}) \tag{3}
\end{equation}

and each succeeding channel has the transfer function

\begin{equation}
G_2 = G_3 = \cdots = \frac{1}{z}. \tag{4}
\end{equation}

Suppose it is desired to simulate the transfer function

\begin{equation}
G(z) = \frac{z(a_0 + a_1)}{z^2 + b_1 z + b_2} = E_n^*. \tag{5}
\end{equation}

Solving for $E_0^*$,

\begin{equation}
E_0^* = a_0 E_n^* + \frac{a_1 E_n^*}{z} - \frac{b_1 E_0^*}{z} - \frac{b_2 E_0^*}{z^2}. \tag{6}
\end{equation}

The block diagram for this is shown in Fig. 6. The implementation of a second-order difference equation can be accomplished with three sample-and-hold channels instead of four; the diagram in Fig. 6 was chosen because it made the error analysis more expedient.

**Accuracy of Simulation**

Each Addaverter unit (ADC or DAC) is accurate to within ±0.1 per cent of its nominal input (or output) voltage. Therefore, the voltage out of each sample-and-hold channel is accurate to within 0.2 per cent of the voltage put into the channel. Because of the 0.2 per cent inaccuracy of the Addaverter, some uncertainty in the solution of a difference equation may arise. The following discussion treats the limitations caused by this inaccuracy.

A sample-and-hold channel may be visualized as having its output made up of the sum of a voltage which is identical to the voltage at the input during the last sample and an unknown random voltage. This is graphically explained in Fig. 7.

Using Fig. 7 as a model sample-and-hold channel, the complete diagram for a second-order difference equation is shown in Fig. 8.

The expression for the output is

\begin{equation}
E_0^* = E_n^* G(z) + R_1^* G(z) + R_II^* \frac{a_1}{1 + \frac{b_1}{z} + \frac{b_2}{z^2}} - \frac{b_1 + \frac{b_2}{z}}{1 + \frac{b_1}{z} + \frac{b_2}{z^2}} \tag{7}
\end{equation}

Further analysis would be impossible without some simplifying assumptions about the character of $R_i$. An $R_i$ is a function of the voltage at the channel input and hence cannot be assumed Gaussian. Observations taken from the Addaverter have shown that the $R_i$ are, to a first approximation, constant offsets. Therefore, let

\begin{equation}
R_i^* = \frac{z}{z - 1} \epsilon_i \tag{8}
\end{equation}

where $\epsilon_i$ = constant offset associated with the $i$th channel.

Furthermore, to make analysis expedient, the second-order difference equation is assumed to be that of a damped sinusoid:

\begin{equation}
G(z) = \frac{z e^{-\alpha T} \sin aT}{z^3 - 2z e^{-\alpha T} \cos aT + e^{-2\alpha T}}. \tag{9}
\end{equation}
Substituting (8) and (9) into (7),

\[ E_0^* = E_0^* G(z) + \frac{z}{z-1} e_1 G(z) \]

\[ + \frac{z}{z-1} e_{11} \frac{z^2 e^{-aT} \sin aT}{z^2 - 2z e^{-aT} \cos aT + e^{-2aT}} \]

\[ + \frac{z}{z-1} e_{11} \frac{2z e^{-aT} \cos aT - e^{-2aT} e_1}{z^2 - 2z e^{-aT} \cos aT + e^{-2aT}} \]

\[ - \frac{z}{z-1} e_{11} \frac{e^{-2aT} e_2}{z^2 - 2z e^{-aT} \cos aT + e^{-2aT}} \] (10)

The first term of (10) represents the desired output of the simulation. The remaining terms represent the error caused by the offsets in the channel outputs. The error is split into two parts: a constant term and an oscillatory term. These two parts are evident in the partial fraction expansion of the error terms of (10).

The first term of (10) contributes a constant error, which would be most noticeable if \( aT \) were small. If \( a = 0 \), the error due to the first term would be

\[ \text{error} = \frac{(2e^{-aT} - e^{-2aT})e_{11} - e^{-2aT} e_1}{1 - 2e^{-aT} + e^{-2aT}}. \] (12)

Suppose \( e_{11} \) and \( e_{11} \) are offsets which are of such probability that they give the largest error: namely, suppose \( e_1 = +\psi \) and \( e_1 = -\psi \) where \( \psi \) is the average channel offset. Furthermore, suppose the total error of simulation is to be no larger than 100 \( \psi \) (i.e., the error-to-offset ratio is 100).

Then substituting into (12)

\[ 100\psi \geq \frac{2e^{-aT}\psi}{1 - 2e^{-aT} + e^{-2aT}}. \] (13)

Solving for \( e^{-aT} \),

\[ e^{-aT} \leq 0.868. \] (14)

\( e^{-aT} \) is the \( z \)-plane location of the double pole for the case when \( a = 0 \). Hence, if it is desired to have an error no greater than 100 times the average channel offset, then the real part of the pole locations, for small \( aT \), must be less than 0.868.
Fig. 9—Forbidden region of \( z \) plane for poles of a second-order difference equation.

Now consider the case where \( a = 0 \). The major source of error in the solution comes from the second term of (11). This error is

\[
\text{error} = \frac{\left[ (e_1 + (2 \cos aT - 1)e_1) \sin aT + [(2 \cos aT - 1)(2 \cos aT) - 1]e_{in} - (2 \cos aT - 1)e_{iv} \right] z^2}{(2 - 2 \cos aT)(z^2 - 2z \cos aT + 1)}
\]

By assuming that \( \cos aT \) is approximately unity, and that \( e_{in} = \psi_f \) and \( e_{iv} = -\psi_f \) (as before) and neglecting \( (e_1 + e_2) \sin aT \), the numerator of (15) reduces to

\[
2(z^2 - z)\psi_f
\]

which is the same as the numerator of the transform for \( \cos aTn \). Hence, this error contributes

\[
\frac{\psi_f}{1 - \cos aT} \cos aTn
\]

to the output of the simulation. If as before, it is desired that this contribution be less than 100 times the average channel offset, \( \psi_f \), then

\[
100\psi_f \geq \frac{\psi_f}{1 - \cos aT}.
\]

Solving for \( aT \),

\[
aT \geq 8.1^\circ.
\]

Thus, using (14) and (17), a region of the \( z \) plane can be enclosed, and dubiously named a forbidden region for the existence of poles of a second-order system. Fig. 9 illustrates this forbidden region.

Similar calculations made for an error contribution which is not to exceed 10 times the average channel offset, \( \psi_f \), yield the limits

\[
e^{-aT} \leq 0.641, \quad \text{and}
\]

\[
aT \geq 25.9^\circ.
\]

A similar analysis for a first-order lag may be made by letting \( a_1 = b_2 = 0 \) and \( a_0 = 1 \) in (5). For an error-to-offset ratio of 100

\[
100\psi_f \geq \frac{b\psi_f}{1 - b_1}.
\]

Solving for \( b_1 \),

\[
b_1 \leq 0.99.
\]

Hence, for a first-order lag, the pole should be located at points less than 0.99 for a ratio of 100. This means that this technique of simulating difference equations cannot be used to do integration (the above case, for instance, where \( b_1 = 1 \)).

**CONCLUSIONS**

Simulation facilities possessing general-purpose sampled-data simulators in the form of a conversion system and a digital computer may require an auxiliary sampled-data simulator for simulation of simpler system problems.
An auxiliary simulator which uses the main portions of the analog-to-digital conversion system satisfies most of the requirements that a sampled-data simulator must meet. It has the advantage that it uses the conversion system during otherwise idle periods.

The simulator consists basically of a number of sample-and-hold channels which, subject to certain inaccuracies, may introduce errors into a simulation. The magnitudes of these errors can be determined in cases only with prior knowledge of the complete system.

In the simulation of digital transfer functions, the effects of these inaccuracies on the representation of transfer functions may be analyzed for any given transfer function. Given some bound on the over-all error, a region of existence of the z-plane poles of the transfer function may be outlined.

ACKNOWLEDGMENT

The author is indebted to the members of the Simulation Department of Space Technology Laboratories for their assistance and guidance in the development of the auxiliary sampled-data simulator. Eli Anfenger of EPSCO, Inc., is credited with the original idea for the simulator.

The author is particularly grateful to R. P. Adams, who designed the control logic; E. A. Goldberg, who assisted with the error analysis; and George Chitel, who supervised the construction of all of the additional equipment.

FOXY 2: A Transistorized Analog Memory for Functions of Two Variables

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FOXY 2 is a high-speed function generator with two independent variables for use in analog computers and simulators.

FOXY 2 stands for Function Of X Y, 2nd type. It is a high-speed transistorized version of FOXY 1, which is a relay device.

The mathematical approach is shown in Fig. 1. The x, y plane is ruled in a grid and the value of z at each corner point is defined in the memory.

For any point (x, y) the 4 surrounding corner values $z_A$, $z_B$, $z_C$, $z_D$ are switched to the output circuit, and $z(x, y)$ is obtained by so-called bilinear interpolation as follows: $z'$ is obtained by linear interpolation between $z_A$ and $z_B$, and $z''$ is obtained by linear interpolation between $z_C$ and $z_D$. Then $z$ is obtained by linear interpolation between $z'$ and $z''$.

The memory comprises a jack for each corner point and a set of fixed voltage jacks. To record a value of 47 for $z_{x,y}$ for example, corner jack 5, 9 is wired to voltage jack 47. To provide for wiring several corner jacks to a single voltage jack, the corner jacks are actually twin