for a high-speed converter, the Datrac was also used to digitize analog voltages from telemeter tape at rates of 4,000 words per second.

The 25-channel Epsco Analog-to-Digital and Digital-to-Analog Converter (Addaverter) was delivered untested and not fully debugged in April 1957, and because of modifications suggested by subsequent tests, it was September 1957 before active testing got under way. Until December 1957 the Addaverter was used actively in several different applications such as simulations, checking analog multiplier accuracy, and the digitizing of analog-telemetered data from magnetic tape. During the time from December 1957 to May 1958, the Addaverter was moved to the new astronauts plant and checked out with an International Business Machines 704 in preparation for complete missile simulations.

Combined analog and digital computer simulation is admittedly an expensive method since it requires the use of three costly units of equipment, and at the same time presents the problem of effective utilization of each of the units since considerable skill and maintenance is necessary to make all three operate satisfactorily at the same time. System debugging and checkout time is expected to be held to a minimum by the use of a 704 simulator and by careful checkouts of each of the units before they are interconnected. With the limited experience encountered so far, it seems that efficiency is increasing so that with advances in the state of the art and increased operating proficiency, combined computer simulations will be about 90% as efficient to debug and check out as a purely analog simulation, (which lacks needed accuracy, and about 90% as accurate as a purely digital simulation which is never possible if analog "hardware" is included in the loop. Once debugged, it has been determined that large combined simulations can be set up for re-runs for production faster than all-analog runs of comparable over-all magnitude.

In a combined analog-digital simulation a compromise must always be made between what percentage of a given simulation will be put on the analog computer and what will be put on the digital computer. At the present time an average simulation uses from 500 to 2,000 digital machine instructions and at present digital computer speeds, relatively low sampling rates of one to one hundred per second are necessary to permit this number of digital computations to be made per sampling time increment.

In some simulations where an analog-digital system is being simulated by the Addaverter system the sampling rate is made the same to increase the validity of the simulation. In one such simulation the sampling frequency was 2 per sec. The computations desired were limited and efficient techniques were used to stay within the 500-millisecond computing (ms) time available. The actual computation used required about 400 ms. Thus 100 ms of time was available for further modifications and expansions. In other simulations where the sampling frequency is not fixed by the simulation, it is determined by the amount of time required for the digital computation and usual care must be used in choosing sampling rates so that the conversions will not be made so often that intolerable rounding errors will occur and yet often enough to prevent excessive truncation.

As computer speeds increase in coming years, a greater percentage of each simulation will be made on the digital machines for accuracy reasons. It is conceivable that ultimately only the actual analog hardware in the loop will be handled by analog equipment.

To change viewpoint for a moment, alternate methods of performing accurate, real time simulations should be mentioned. With the development of transistorized and high-speed electronic digital differential analyzers and recently announced compatible high-speed converters, there is indicated the possibility that many small-magnitude combined simulations where extreme accuracy is not necessary may be done more cheaply by using an electronic digital differential analyzer, a small analog computer for making parameter variations and including hardware, and a converter to link the two.

As a final indication of the necessity and importance of combined analog and digital simulation it should be noted that aircraft and missiles have now become so complex that they, themselves, are in many cases controlled by self-contained combination analog-digital systems. To design and develop such systems and to investigate their behavior with superimposed noise, it is essential that combined analog-digital computing equipment at least one order of magnitude better be available to simulate them.

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**Digital Computer Solution of Differential Equations in Real Time**

H. J. GRAY, JR.
NONMEMBER AIEE

Digital computers have been used for some time now to obtain the solutions of differential equations. The first electronic digital computer, the Electronic Numerical Integrator and Computer, (ENIAC)1 was designed and built at The Moore School of Electrical Engineering under contract with the Aberdeen Proving Ground for the specific purpose of integrating the differential equations of motion of a projectile. This computer was very successful at this task. Digital computers have been used to prepare tables of mathematical functions which are solutions of differential equations and one occasionally reads that a digital computer has been used to check, but not in real time, the results of an analog simulation of some system described by differential equations. When

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2 The work reported here is the work of many people whom the author wishes to thank, especially Dr. Morris Rubinfeld, Mr. Herbert Gurk, Mr. Cornelius Eldert, and Dr. F. J. Murray.

3 This work was performed in connection with contracts N60154(24915), N9551(02), and N61339-272, with the United States Naval Training Devices Center, Office of Naval Research, Port Washington, N. Y.
have been used for many years in the training of airplane pilots, a notable example being the Link Trainer. Present-day OFT’s are complex devices employing computers which solve the equations of motion of the aircraft for which the pilot is being trained while the student pilot sits in a cockpit which furnishes the inputs to the computer. The solutions of the equations of motion are modified by further computation to provide the instrument readings and stick forces which are returned to the student pilot. In addition the instructor can send input data to the computer and can receive output data. A block diagram of an OFT computer is shown in Fig. 1 which shows the character of the inputs and outputs.

Nature of the Problem

Before discussing the factors that are related to the choice of computer, it will be attempted to give the reader some further indication of the nature and magnitude of the problem. If the aircraft can be considered as a rigid body one has the equations:

\[
\begin{align*}
\dot{u} &= g X/W - q I_3 - w q + v r \\
\dot{v} &= g Y/W + g m_3 - u r + w p \\
\dot{w} &= g Z/W + q I_3 - w p + u q \\
I_{x_3} \dot{\rho} &= L + (I_y - I_z) q r \\
I q \dot{\gamma} &= M + (I_z - I_x) p \rho \\
I_\rho \dot{\phi} &= N + (I_x - I_y) q p \\
\rho &\dot{=} r m_3 - q n_3 \\
\rho_3 &\dot{=} p m_3 - r l_3 \\
\rho &\dot{=} q l_3 - p m_3
\end{align*}
\]

where \(u, v, w\) are the linear velocities of the aircraft referred to axes imbedded in it; \(p, q, r\) are, similarly, angular velocities; \(I_3, m_3, n_3\) are three of the nine direction cosines relating the airplane axes to ground axes; \(g\) is the acceleration of gravity; \(I_x, I_y, I_z\) are the principal moments of inertia; \(W\) is the weight; \(X, Y, Z\) are the aerodynamic forces; and \(L, M, N\) are the corresponding torques. In addition to these equations, there are equations for \(X, Y, Z, L, M, N\) under a variety of conditions, equations assuring the normality and orthogonality of the direction cosines, equations relating instrument readings with the dynamic variables, etc. The total number of equations for a typical subsonic jet aircraft is such that when typed to a reasonable density, approximately 8\(\frac{1}{4}\) by 11 inch sheets of paper are required. The equations have been programmed for Universal Digital Operational Flight Trainer, (UDOFT) a digital-computer-activated flight trainer designed by the Moore School and now being built by Sylvania Electric Company. This computer will take approximately 11 milliseconds to perform the computations necessary to advance one interval in the numerical integration of the differential equations including all subsidiary computations. For a typical supersonic jet aircraft, the computer takes approximately twice this time, an indication of the amount of computation involved in the case of supersonic aircraft. It is apparent, then, that the problem under discussion is of appreciable magnitude.

The Computer

All of the trainers in existence use analog techniques in order to perform the functions of the block labeled “computer” in Fig. 1. (At the time of the writing of this paper UDOFT had not yet been put into operation.) Certain reasons have been put forth giving the desirability of using digital techniques. (It should be mentioned that the number of analog inputs and outputs to the computer in Fig. 1 was found to be equal to the number of discrete or digital inputs and outputs required in the case of the simulation of the afore-mentioned aircraft. Therefore, the question of compatibility of computer type with inputs and outputs does not arise.) Foremost of these reasons is that a digitally computed flight solution is as accurate as the numerical method used, and analog computer problems such as amplifier drift, errors due to component drift, etc., do not arise. Other advantages of digital techniques will be stated later.

Digital Techniques

Two types of digital machines were considered at first, the digital differential analyzer (DDA), and the digital computer of the general-purpose stored-program kind with, possibly, some modifications to make it better suited to the problem. At that time the only existing DDA was the MADDIDA, which was too slow. In addition, a hypothetical DDA using the best of digital computer techniques also proved to be too slow. The main reasons were that the method of integration required a small integration interval to get good results, addition using integrators is essentially equivalent to counting, and much of the computation is essentially serial in nature. On the other hand, a hypothetical digital computer using the same digital computer techniques, while still too slow, showed more promise of success. This led to the development of the UDOFT. The characteristics of UDOFT are as follows:

Address structure: single address
Number representation: signed 20-bit magnitude
Information rate: 1.2 millicycles per second
Order memory: 4,095 words, 20 bits per word
Number memory: 4,094 words, 22 bits per word
Input: punched card reader used to load problem
Output: Teletypewriter printer available for test purposes

Discrete inputs: 64 toggle switches
Analog inputs: 24 Gray code wheels
Analog outputs: 64 multiplexed signals
Digital outputs: 24 on-off signals
Operation times: addition, subtraction, and similar operations, 5 microseconds; multiplication, 10 microseconds; divide, 105 microseconds.

In addition to high accuracy as a reason for the use of digital techniques, there are several additional reasons for the use of a stored-program digital computer. These are derived from the characteristics of such computers and are as follows:

1. A digital computer is flexible and can be changed by programming from the simulation of one plane to another. This flexibility also makes it possible to change flight conditions during a test run or to alter aerodynamic coefficients gradually in order to test the effects of such changes on the flight of the plane and the response of the pilot.
2. One digital computer should be able to solve the equations for several cockpits simultaneously, allowing a group of trainees to receive simultaneous instruction, either in independent flights, in flight formations, or in simulated combat.

Comparison of Digital and Analog OFT’s

A summary comparison of digital and analog OFT’s will now be made on the basis of accuracy, speed, flexibility, ease of maintenance, and cost.

Fig. 1. Block diagram of an OFT computer

From the collection of the Computer History Museum (www.computerhistory.org)
ACCURACY AND SPEED

The analog OFT's, operating as they do in real time, have been found accurate enough to satisfy the desires of experienced pilots in most cases. Accuracy of results for an analog machine depends on the degree to which the various components of the computer approximate the functions which they are simulating.

The digital OFT is as accurate as the numerical method used. Actual calculations of simulated maneuvers have been carried out using the methods of the digital OFT. These have given quite accurate results. A mathematical theory has been developed enabling the prediction of the maximum interval permissible in the numerical solution of the differential equations. This interval is several times larger than the time it takes UDOFT to perform the calculations. The mathematical theory will be discussed shortly.

FLEXIBILITY

Proposed analog computers would be able to simulate any one of a number of airplanes, although existing types would essentially have to be rebuilt to do this. The proposed computers would use plug-boards for different planes in order to change numerical values (i.e. resistances) and mathematical relations (connections). A number of different plugboards would be needed for each plane.

For a digital OFT, the airplane to be simulated could be changed by reprogramming the computer. As with the plugboards, the programming would have to be prepared in advance.

The cockpit connections in present analog OFT's are often tied directly to the computer, in some cases being mechanical connections. Such connections might cause some difficulty when the planes being simulated are changed. For a digital OFT, all connections are made via multiwire cable connector. One would anticipate, however, that this could be done in the proposed analog OFT's.

A digital computer, if it is fast enough, can solve the equations of more than one aircraft. The arithmetic unit of UDOFT is fast enough for at least three aircraft and there will be even faster digital computers in the future. It is true, however, that the UDOFT design allows only enough memory space for one aircraft.

To simulate more than one aircraft on an analog computer requires the addition of sufficient analog components to make more than one analog computer. Such a simulation seems to be quite impossible, at present, on a single analog computer. Thus simulation of two aircraft would require two analog computers.

EASE OF MAINTENANCE

The components of both the analog and digital computers gradually drift or deteriorate. In an analog computer this gradual change in components shows up as a gradual change in results. In a digital computer, however, such changes in components have no effect on the accuracy of results until some component has deteriorated substantially. Then the error is discrete and immediately noticeable. Thus, it is generally easier to detect the malfunctioning of a digital machine than that of an analog machine.

COST

OFT's, both analog and digital, are, at the present time, quite expensive pieces of equipment. Modular construction of the digital computer and mass production of modules may make the digital OFT competitive in cost with the
analog OFT. Existing data, while meager, show the digital OFT to be somewhat more expensive than the existing analog OFT's. One might expect however, that the proposed analog OFT's, being more complex, would be more expensive than the existing analog OFT's.

**Numerical Methods**

Success of a digital OFT and some of its superiority are derived from the quality of the numerical methods used and the ability of the mathematical theory to predict the accuracy of the simulated behavior of the aircraft. It is the numerical method together with a fast computer that makes the digital computer solution of differential equations in real time possible. One would expect to see in this "space age" more and more need for the real-time solution of differential equations and for these reasons a brief exposition of the mathematical theory follows.

**Quadrature Methods**

In the numerical solution of the differential equation

\[ \dot{x} = f(x,t) \]  

(2)

one common way is to approximate the value of \( x_n \) by a formula of the type \( O_{NM} \), called an "open" formula

\[ x_n = \sum_{j=1}^{N} a_j x_{n-j} + h \sum_{j=0}^{M} b_j x_{n-j} \]  

(3)

from a sequence of values of \( x \):

\[ x_{n-3}, x_{n-2}, x_{n-1} \]  

where \( x_0 \) and \( x_1 \) are respectively the values of \( x \) and its derivative, computed from equation 1, at \( t = jh \) (\( j \) an integer), \( h \) is the quadrature interval (a constant), and \( a_j, b_j \) are the known coefficients of the quadrature formula. This yields a new sequence...

\[ x_{n-5}, x_{n-4}, x_{n-1} \]

which, together with equations 2 and 3 enable one to obtain \( x_{n+1} \), \( x_{n+2} \), etc. In this way the solution of the differential equation is approximated by the values of \( x \) at times separated from each other by multiples of \( h \).

The coefficients in the formula given by equation 3 can be obtained by the so-called polynomial method, which for a given formula is equivalent to choosing the coefficients so that the positive integer \( R \) is a maximum, where the equations \( \dot{x} = 0, \dot{x} = 1, \dot{x} = t, \dot{x} = t^2, \dot{x} = t^3 \) are solved exactly by the quadrature formula. This procedure involves the solution of a set of simultaneous algebraic equations. The formula, \( O_{NM} \), obtained in this way, is

\[ x_n = -4x_{n-1} + 5x_{n-1} + h(4x_{n-1} + 2x_{n-1}) \]

When the coefficients are obtained in this way, the formula is called "classical".

If the formula makes use of \( x_0 \), it is called a "closed" formula, \( C_{RQ} \). Initially, a guess at \( x_0 \) must be obtained, usually by an open formula. Equation 4 is used repeatedly to generate iteratively over one interval successively better values of \( x_n \) before proceeding to the next interval. This is called "method \( rC_{RQ} \)." An example of a classical closed formula is \( C_{12} \), Milne's formula.

\[ x_n = x_{n-1} + h \left( \frac{1}{3} x_n + \frac{4}{3} x_{n-1} + \frac{4}{3} x_{n-2} - \frac{1}{3} x_{n-3} \right) \]  

(4)

A third method uses an open formula, \( O_{NM} \), to estimate \( x_n \) and follows it by a single application of a closed formula, \( C_{RQ} \). Such a method is called a "mixed method" and is denoted by \( [O_{NM}, C_{RQ}] \). A classical mixed method uses classical \( O_{NM} \) and \( C_{RQ} \) formulas.

Application of any of these methods to equation 1 requires the evaluation of the right-hand sides of each of the equations once in an interval for open and mixed formulas, compared to many such evaluations in an interval for any \( rC_{RQ} \) method. Hence, the \( rC_{RQ} \) methods are not suited for real-time computations.

**Stability Charts**

In the use of any of the open, closed, or mixed formulas to solve differential equations in real time, it is necessary to have some way of comparing the asymptotic behavior of the computed and true solutions. (Asymptotic behavior is of importance because of the long duration of the computation.) This is accomplished by means of "stability charts."

If the differential equation

\[ \dot{x} = \lambda x \]  

\( \lambda \) complex

is solved analytically, its solution is found to be

\[ x = x_0 e^{\lambda t} \]

The real part of \( \lambda \) determines the rate at which the solution grows or decays and its imaginary part is \( 2\pi \) times the frequency of the oscillations in the true solution. Hence \( \lambda \) is a natural frequency of equation 5.

When equation 5 is numerically solved using any of the quadrature formulas mentioned, a set of points result which, it has been proven, can be fitted exactly by

\[ \sum_j c_j x_j \]

provided the correct number of terms and the correct values of the \( c_j \) are chosen. The \( c_j \) are the natural frequencies in the computed solutions. The asymptotic behavior of the computed solution is determined by the \( c_j \) having the largest real part. If this \( c_j \) is denoted by \( c_1 \), then the computed solution is very nearly given by

\[ x_n = c_1 e^{Z_1} \]

It also has been shown that if \( c_1 \) is close to \( \lambda \), then \( c_1 \) is close in value to \( \lambda \). Therefore, if \( c_1 \) is close in value to \( \lambda \), the computed solution of equation 5 will be close to the true solution of equation 5. A stability chart is a picture showing how \( c_1 \), a complex number, is related to \( \lambda \), also a complex number.

A stability chart for classical \([O_{20}, C_{12}] \) is shown in Fig. 2. It has been defined that

\[ z = x + iy = \Re \lambda \]

\[ w = u + iv = \Re \mu \]

The use of the chart is best shown by an example. If the equation \( \dot{x} = (-2 + 2i) x \) is solved by this formula and a step of \( h = 0.1 \), one has \( z = -0.2 + 0.3 \). This yields a value \( w = -0.19 + i0.29 \) read from the chart or \( \mu = -1.9 + i2.9 \). Whether or not this is satisfactory depends on the nature of the problem. However, if \( h = 0.2 \), \( z = -0.4 + i0.6 \), \( w = 0.26 + i2.19 \), and \( \mu = 1.3 + i11 \) it is clearly unsatisfactory.

The validity of the application of stability charts has been extended rig-
orously to linear systems having several natural frequencies and to linear systems containing forcing functions. As yet no rigorous proof exists extending the validity of the stability charts to varying-parameter linear systems and to nonlinear systems. However, experimental computations have shown that in every case tried, the stability charts have successfully predicted the behavior of the computations. This is not too surprising in the case of not too severe 2-dimensional aircraft maneuvers because the equations of motion are not greatly nonlinear. However in the case of a particularly violent maneuver, the Immelman turn, results were surprisingly good as predicted. In this maneuver, the aircraft makes half an inside loop followed by a roll through 180 degrees, to that the aircraft finishes the maneuver right-side up. The violence of this maneuver is indicated by the fact that there was a maximum acceleration of 7 g during the inside loop.

Computations were first made using a small interval with classical $rG_1$ to provide a reference standard. Computations were then repeated using two nonclassical methods, $O_B$ mod Gurk, and $[O_{2C}, C_3]$ mod Gurk, using an interval of 50 milliseconds. The yawing velocity, $r$, is plotted in Fig. 3. The greatest error appeared in this variable, yet the results are quite acceptable for simulator use. Note that the results asymptotically approach the reference standard and differ only during the transient oscillations.

**Expansion in Series**

The frequency $\mu_1$ is clearly a function of $\lambda$. Under certain conditions it is possible to obtain a power series expansion of $\mu_1$ in terms of $\lambda$. In the case of classical $rC_{N+M}$ formulas, a particularly interesting result is obtained as follows:

$$w = \mu_1 z + \frac{1}{N+M} \sum_{l=M+1}^{N} \left[ \frac{N(M-1)!}{(N+M-1)!} \right] z^l$$

where $z = \lambda \beta$.

Tables of the coefficient of $z^{N+M}$ have also been obtained for $N < M - 1$ and are fairly straightforward to obtain for the open formulas also. Equation 6 is useful in estimating $\mu_1$ when more accuracy is required than is afforded by the stability chart.

**Synthesis of Nonclassical Methods**

The complex frequencies, $\mu_1$ and $\lambda$, are related by a function, $z = f(w)$, for open, closed, and mixed methods. The characteristics of this functional interdependence are determined by the coefficients of the quadrature formula which also appear in the function, $f(w)$. Hence, the characteristics of the stability chart are also dependent on the values or these coefficients. From knowledge of the type of functional relationship between $z$ and $w$, it has been possible to develop some general tools useful for synthesis of completely new formulas having good stability charts. (A good stability chart is one where in a large region around the origin the function $z = f(w)$ approximates the identity mapping, $z = w$, hence $\mu_1 = \lambda$.) One synthesis method is to impose less than the maximum number of conditions required for the determination of the coefficients by the polynomial method and then to choose the remaining coefficients in such a way as to maximize the good region. The greatest success, however, has been obtained by using what is called the “shifting technique.” Often, a classical or mixed classical method has a stability chart which exhibits a good region extending into the right-half plane. An example of such a method is classical $[O_{2C}, C_3]$ shown in Fig. 2. If the chart were shifted such that the $u = 0.2, v = 0$ point coincided with the $z = 0$ point and were renamed $w = 0$, an excellent quadrature method for real-time computations would result. The mathematics to do this is quite simple and the resulting stability chart has yielded the two nonclassical methods $[O_{2C}, C_3]$ mod Gurk, and $O_B$ mod Gurk.

**Conclusions**

At the present time digital computers seem to be more than competitive with analog computers and digital differential analyzers in the real-time solution of differential equations. UDOFT, for example, is more than adequately fast for its application. The flexibility of a digital computer is also of great value in the OPT problem. For example, much of an analog OPT is involved in operations other than the solution of the equations of motion of an aircraft. These “pinball machine” operations can be and are going to be performed by the digital computer. There appears to be much that can be done with digital computers in real time and it is hoped that this illustration of an application will stimulate more investigation.

**References**


Gray—Digital Computer Solution of Differential Equations

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Discussion

Walter W. Varner: I have one afterthought on the Addavertor. I pointed out in my talk that it is quite difficult to get the three pieces of machinery working at the same time. I did neglect to mention that one of the other great difficulties is the fact that there are people in this loop, and getting people to work with these three pieces of equipment; that is, scheduled, and so forth, is also quite difficult.

I do not want to create the impression that we were able to predict the error exactly, but we were able to estimate the error well enough to determine whether or not the solution was good enough to meet the requirements of people who posed the problem for the machine.

Chairman Madden: A question for Mr. Gray from M. K. Haynes, IBM Corporation: "What method and formula for quadrature have you found to be optimum, and exactly what is the formula used in UDOFT?"

H. J. Gray: So far we have found optimum, a method called $O_{33}$ Mod Gurk, and it is the equivalent (this method) to $[O_3, C_3] \mod$ Gurk. Our optimum method gives us the largest region around the origin which is good. This method is obtained from the one shown in the paper, by shifting the point $u$ equals 0.2 to the origin, and the mathematics for finding the coefficients is quite straightforward.

Chairman Madden: A question for Mr. McLeod by J. Murphy: "In your discussion you mentioned the resolver which produced a rapid answer to parts of your formula. Could you explain how the resolver works?"

John McLeod: For the purposes of this discussion the resolver is a black box which has two inputs and two outputs. In one mode of operation the input may be the slant range $R$ of a radar target and gamma, the elevation angle. The outputs would then be $R \cos \gamma$, the surface distance, and $R \sin \gamma$, the elevation of the target. Conversely, in the other mode of operation, the surface distance $R \cos \gamma$, and the elevation $R \sin \gamma$ may be the inputs, in which case the outputs would be the elevation angle gamma and the slant range $R$. As was indicated, in the most popular kind of resolver, the voltage representing gamma will position a servo driving one or more sine-cosine potentiometer across which the voltage representing $R$ is applied.
Switching Transistors

I. M. Ross
NONMEMBER AIEE

Transistors are being used more and more frequently as components of computing systems. This increasing popularity can be attributed to their small size, high efficiency, and potential reliability. The size and efficiency of transistors are sufficiently well known to require no further discussion here.

On the reliability question, it is pertinent to note that transistors have been in operation in some telephone applications for several years with failure rates of about 0.05% per thousand hours. This figure compares favorably with that for the best vacuum tubes and there is every confidence that newer devices will exhibit much higher reliability.

The first part of this paper is devoted to a discussion of the electrical characteristics that make conventional transistors (n-p-n and p-n-p) attractive in computing applications. The second part deals with a family of 4-region (p-n-p-n) devices which are now under development and in some cases in early production. These devices exhibit a bistable characteristic and their use may lead to a considerable simplification of computer circuitry.

Conventional Transistor Switches

A common form of switching transistor is shown in Fig. 1. Because such a device is frequently made by alloying techniques, it will be referred to as an "alloy-type" transistor. One essential feature of such a structure is that the emitter and collector regions are metallic and hence, do not introduce appreciable series resistance in the emitter and collector leads. A further feature is that the emitter and collector junctions are opposite one another and are nearly the same size, the collector usually being somewhat larger than the emitter. Thus, the device is close to being symmetric.

Because of the lack of series resistance in the collector and emitter leads and also because of the symmetry of the device, this transistor is probably the most suitable for switching applications. It is further the most amenable to accurate analysis.

The simplest form of switching circuit using an n-p-n transistor is shown in Fig. 2. If the base current is very small or even negative, the current through the load resistor is of the order of the leakage current across the reverse biased collector junction and is microamperes or less. Thus, effectively all the supply voltage appears across the transistor which therefore acts like an open switch.

When a current \( I_b \) flows into the base lead, a current \( I_{0n} / (1 - \alpha) \) will flow in the collector circuit. The collector voltage approaches zero as the collector current \( I_{0n} / (1 - \alpha) \) approaches the magnitude of the collector forward bias. Effectively, all the supply voltage then appears across the load resistor and the transistor behaves like a closed switch. If the base current is increased beyond this critical value, the current that would be collected by a reverse biased collector becomes greater than the current that can be supplied through the load resistance from the voltage source. In fact, the collector junction becomes forward biased in order to reject or reinject some of the current and maintain the collector current close to the limiting value determined by the external circuit. Thus, for large values of base current, both the emitter and collector junctions are in the forward biased direction and the emitter to collector voltage, which is the difference of the two forward biases, becomes very small, as small as a few millivolts. This condition of operation with both junctions forward biased and the collector current saturated is referred to as the saturated condition. It should be noted that in the saturated condition, the emitter to collector voltage is less than the emitter to base voltage by the magnitude of the collector forward bias. Thus, if the collector of a saturated transistor is directly connected to the base of a second transistor the second transistor will be maintained in its off condition. The use of this mode of operation can lead to very simple switching circuitry.

A family of collector characteristics with base current as the parameter is shown in Fig. 3 for a silicon alloy-type transistor. For d-c operation, the most important parameters are:

1. The leakage current in the open or off condition. This depends on the dimensions of the device, the method of fabrication and most critically, on the material from which the device is made. For germanium transistors, the leakage current is of the order of microamperes at room temperature and doubles approximately every eight degrees Centigrade with increasing temperature. For silicon, the leakage currents are of the order of milli-microamperes and the temperature dependence is somewhat smaller than in germanium. For this reason, silicon transistors are preferred over germanium in many switching applications.

2. The "breakdown" voltage in the off condition. As the applied voltage approaches this value, the leakage current increases rapidly and the device no longer represents an open switch. Transistors can be designed to have breakdown voltages as high as 50 to 100 volts.

Ross—Switching Transistors

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