Electronic Differential Analyzers in Perspective

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IN THIS paper's contribution to the discussion of Contrasting Tools and Techniques for Simulation, "Electronic Differential Analyzers in Perspective", the emphasis will be on perspective. There is no other field of technical endeavor in which the picture is so often thrown out of focus and distorted by point of view as that which this paper will attempt to clarify. It is not difficult to realize why this is true. The field of simulation, dependent as it is on both the technique and equipment of machine computation, is comparatively new and growing fast; so fast, in fact, that even those who have been in it for some time are unable to keep up with all developments. And people who have become involved more recently have usually been so busy trying to apply the tools available to the problem at hand that they have scarcely had time to look over the other fellow's shoulder. Hence this discussion.

So that others will understand the author's comments, even if they do not agree, simulation will be defined as the use of electronic analog and/or digital computers to determine the dynamic behavior of physical systems. The computer mechanism may or may not operate on a one-to-one time ratio with the physical system being simulated, and it may or may not include parts of the physical system. Other methods of simulation are recognized but will not be discussed here.

Simulations of the kind under consideration may be mechanized on direct-analog, electronic differential analyzer, digital differential analyzer, general-purpose digital, or special-purpose computers, or combinations of these. And, although the choice of which to use is often dictated by availability than suitability, the assumption will be made here that suitability is the only factor of interest.

The remarks in this paper will be confined to describing the characteristics, both good and bad, of analog computers (as the author sees them), and rude comparisons with other computers which may sometimes be coaxed into something similar to real simulation will be avoided.

The term "analog computer" refers to a group of "operational amplifiers" and related equipment suitable for solving sets of ordinary simultaneous nonlinear differential equations. Operational amplifiers are capable of accepting several voltage inputs, multiplying them by constant integers, and summing and/or integrating the result with respect to time. They are usually associated in an analog computer with a stable reference voltage and power supply, a number of potentiometers for "multiplying" by fractions, multipliers for multiplying by variables, resolvers for changing co-ordinate systems, function generators for introducing empirical relations, switches, relays, etc. for controlling, and meters for monitoring operation, and finally recorders and plotting boards to record the results. All of these components operate on voltages, which are made analogous to the variables of the physical system. In the general-purpose analog computer the components are connected together by means of patch-cords much as telephones were before the advent of automatic exchanges.

For the benefit of those who are not familiar with the setup of analog computers (and experience has shown there are many too many), a simplified example will be presented to emphasize what the author believes to be the most important characteristics of these computers.

Assume that the effect of changing various parameters of a short-range rocket is to be studied in order to arrive at an optimum design. To begin, assume a constant weight and a constant thrust for a definite duration of time and use the basic equation:

\[ F = ma \] (1)

Solve for the highest derivative,

\[ a = F/m \] (2)

and define the terms:

\[ a = \text{ acceleration along the flight path} \]

\[ V = \text{velocity} \]

\[ S = \text{total displacement} \]

Fig. 1. Double integration of acceleration to obtain displacement

\[ F = \text{summation of longitudinal forces acting on the rocket} \]

\[ m = \text{total mass of the rocket} \]

For the time being consider that the thrust and the aerodynamic drag are large compared to the weight of the rocket, i.e.,

\[ F = F_D + F_T \]

It has been stated that the thrust will be a constant, \( T \), until the cutoff time \( t_c \), and then it will be zero, so that:

\[ F_T = T \quad (0 < t < t_c) \]
\[ = 0 \quad (t \geq t_c) \]

It is known that the drag force, \( F_D = -1/2 \rho V^2 S C_D \) where

\[ \rho = \text{atmospheric density} \]

\[ V = \text{velocity of the rocket along the flight path} \]

\[ S = \text{the equivalent frontal area of the rocket} \]

\[ C_D = \text{the drag coefficient} \]

Now, if this is a short-range subsonic rocket little error is introduced by letting

\[ a = \frac{(T + kV^2)}{m} \] (3)

Now note how straightforward the mechanization of a problem on an analog computer is. Basic to the procedure is what, at first, may seem an invalid assumption. This is that the voltage analogous to the value of the highest-order derivative, in this case, \( a \), can be made available when it is needed.

Therefore in Fig. 1, assume that somehow a voltage can be generated analogous to the acceleration, \( a \). If so, this voltage can be integrated with respect to time in the integrating amplifier (1) to obtain a voltage analogous to the rocket velocity \( V \). If desired, it is possible to integrate that voltage in amplifier (2), to get a voltage which will be proportional to the distance, \( S \), traveled along the flight path.

Now, consider validating that original assumption. What is needed to generate a voltage analogous to \( a \)?

Looking again at equation 3, it is seen that \( T \) is needed as a function of time; therefore, one constant voltage proportional to \( T \), and another voltage which increases proportionally with time are required. These voltages are obtained from the computer reference.

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voltage \( v \). For \( T \) this is supplied through a relay (3) to the scale-factor potentiometer (4) which adjusts it to some convenient value, proportional to \( T \). For \( t \), the reference voltage \( v \) is adjusted by means of the potentiometer (5) to some convenient value, \( i \), and then integrated in amplifier (6) to give a voltage, \( t \), which increases linearly with time. The normally closed relay (3) is set to open when \( t = t_c \), which operation cuts off the simulated thrust.

Referring once more to equation 3, it is seen that a voltage proportional to \( k V^2 \) is also required. This can be obtained as shown by multiplying \( V \), which is already available from amplifier (1), by itself in the multiplier (7) to produce \( V^2 \) which can then be multiplied by \( k \) in pot (8) to give a voltage proportional to \( kV^2 \). Now, adding this to \( T \) in the summing amplifier (9), and multiplying by \( 1/m \) in pot (10) will give \( T + kV^2/m \), which is equal to \( a \). So the assumption that \( a \) could be made available was justified, and the computer is connected to solve the required simple nonlinear second-order differential equation.

Heretofore, the author has referred to velocity and direction along the flight path. Usually, however, one is interested in the shape of the trajectory, and this re-
quires that the direction of the flight path, or flight-path angle, \( \gamma \), be known as a function of time. This can be found by writing the basic centrifugal force equation, \( F_c = \omega^2 m \sin \gamma \), in the form

\[
\dot{\gamma} = \frac{F_n}{mV}
\]

where \( \dot{\gamma} \) is the rate of change of flight-path angle (equal to \( \omega \)), \( F_n \) is the summation of the forces acting normal to the flight path, \( m \) is the mass, and \( V \) is the instantaneous linear velocity of the rocket.

Thus, if one considers a “no-lift” trajectory (one in which there are no aerodynamic forces acting normal to the flight path) the component of gravity normal to the flight path, \( g \cos \gamma \), will be the only force turning the rocket. Now the relationship \( \dot{\gamma} = g \cos \gamma/mV \) can be written and mechanized to find \( \gamma \) by adding the computing components as shown in Fig. 4.

If an \( x-y \) plot of the trajectory is desired, it can be obtained by adding components as shown in Fig. 5 to mechanize the relationships \( V_x = V \cos \gamma \) and \( V_y = V \sin \gamma \), and integrating them to obtain and record \( x \) and \( y \).

If it is desired to examine the rocket performance in more detail the simulation can be made more detailed. As shown in Fig. 6, if the rocket weight is not small compared to the thrust and drag, a term \( g \sin \gamma \) must be added to the acceleration. This term is already available from the resolver which produced \( g \cos \gamma \). If the rocket is high-speed, so that \( C_D \) changes nonlinearly with velocity, and long-range so that the altitude, and therefore \( \rho \), changes considerably the drag force, \( F_D = 1/2 \rho V^2 SC_D \), must be generated by adding the function generators and multipliers as shown, and resetting pot (5) to take care of the only remaining constants \( 1/2 \) and \( S \). Computing elements can as easily be added to take care of other effects of interest, such as the change in mass as fuel is consumed. The value of any of the variables in the system, which are represented by voltages generated in the computer, can be plotted out against time or against other variables, as desired.

Now in this problem only the two-dimensional translational equations in the pitch-plane have been considered. When the three dimensional systems are considered, and rotational and cross-coupling effects are included, the mechanization can, and does, get complicated. Nevertheless, this simple example illustrates one of the most important characteristics of the analog computer: An engineer can build with computer components, just as with system components, a system as simple or as complex as may be required for his purpose. Furthermore, as he builds up the simulation he can maintain a one-to-one correspondence between computer components, or groups of components, and system components, groups of components, or functions, as he sees fit. Thus the analog computer can be made a recognizable model of the physical system under study, and a powerful and flexible experimental tool is created.

Some advantages are obvious:

1. Changes in design parameters can be simulated by twisting a dial, or at most by changing a function in a function generator. For instance, in the previous example, the effect of increasing thrust can be observed by increasing the setting of pot (4). Or the effect of aerodynamic design can be studied by changing the \( C_D \) function generator.

2. Changes in basic design can be simulated by changing patch-cords to change the basic model. Perhaps one would like to know what would happen if some fins were put on the rocket in the example. This would require additional computer components to generate voltage proportional to the lift and drag forces created by the fins. These voltages would then be plugged into the \( \alpha \) and \( \dot{\gamma} \) summing junctions.

3. The engineer is forced to become intimately familiar with the operation of the physical system. For this reason it has been said that this alone would justify the use of an analog computer, even if it never actually “solved” problem one!

Other advantages of the analog computer are the speed with which it can operate and the “language” which it speaks.

The speed of the analog computer results from its operating in parallel with respect to time, i.e., it operates on all of the voltages at the same time, adding, integrating, multiplying, etc., simultaneously. The real importance of this speed is not merely a saving in the operating time necessary to solve a problem, though this is sometimes important, but stems from the fact that it allows the computer to operate in “real” time, i.e., in a one-to-one relationship with the physical system. This not only makes the model more realistic, but is absolutely necessary if components of the actual system are to be used in the simulation, a practice which is sometimes desirable, or, if the response of the system components is in question, necessary.

Advantages of the analog language...
usually accrue to the electronic differential analyzer at both the input and output. Most of the inputs represent quantities in nature, which in general are analog, that is, continuous, rather than discrete. Therefore no “translation” is necessary. At the other end, the output is usually for human consumption, and as most engineers are human and prefer graphs to numbers for the analysis of dynamic behavior they understand the computer. Rapport is established!

So much for the advantages of electronic differential analyzers; have they no shortcomings? They have!

The one you will probably hear talked about most is their lack of accuracy. The author believes this is overemphasized. True, although most individual components will produce errors of only about one part in a thousand these may build up in a complex problem to 2 to 5% or more. However, analog computers are usually used to simulate hardware the performance of which is not predictable, or measurable, with accuracy that great and often not as great. Furthermore, computer errors build up in the simulation in the same way that hardware and measurement errors do in the physical system, so for simulation, the analog computer is usually as accurate as it has to be.

A more serious difficulty is that of answering the question “How does one know he is right?” Qualified answers may be obtained by incorporating some of the following procedures.

1. Component checks.
2. Static checks.
4. Comparison with the physical system.
5. Engineering judgment.
6. Clairvoyance.

Usually some combination of the foregoing is used to build up the necessary degree of confidence. However, checking is still a problem, and is recognized as such by the computer manufacturers who have made, and are making, progress in alleviating it.

One inherent characteristic which may or may not be a disadvantage, depending on the problem, is the inability of the electronic differential analyzer to integrate with respect to any variable other than time. In simulating most physical systems this is no handicap because it is only necessary to integrate with respect to time. In other cases, if it is only necessary to integrate with respect to some other one variable, the problems can be set up in such a way as to represent that variable in the system by time in the simulation.

Another inherent characteristic of the analog computer gives rise to two difficulties. This is the fact that an analog computer setup grows in complexity and number of components with the complexity of the physical system being simulated. Therefore cost goes up and reliability goes down. There seems to be no way around this!

As to the cost, a rule of thumb is that on the average three-operational amplifiers will be used for every order of the set of differential equations being mechanized. This may vary from a little more than one to ten or more (for problems involving a large amount of algebra). Unless the type of problems to be solved is defined, three per degree is about as good a guess as can be made.

The price of an analog computer will vary from about $700 per amplifier to perhaps ten times that amount. The lower figure includes a minimum of auxiliary equipment as contrasted to the more elaborate computers with a complete complement of nonlinear, input-output, and special checkout equipment. The cost of the average installation will probably run between $2,000 and $3,000 per amplifier, or from $5,000 to $10,000 per order of the set of equations. Circumstances may alter these figures by an order of magnitude in either direction, but if one must guess without additional information this is about as accurate as possible.

The foregoing would seem to indicate that if one is willing and able to spend the money he can simulate systems of any complexity. Not so! As stated, the reliability of an electronic differential analyzer, as with anything else, goes down with the complexity of the mechanization. Thus, a point is reached when the equipment simply cannot be made to give useful answers. The author cannot tell where this point is, because it depends on the problem, the equipment, the opera-
tor, and the maintenance man. And, as two humans are involved in the operation of this equipment, the author will not even hazard a guess!

Some people are cowed by the mere idea of an electronic computer, while others can successfully manage complexes of several hundred amplifiers and auxiliary equipment. Moral: pick operators and maintenance men with the same care with which equipment is chosen, and pay them accordingly!

The Case for Combined Analog-Digital Simulation

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The idea of combining the good features of both analog and digital devices is certainly not a new one. Numerous examples can be found in history dating back to the ancients who kept time by a slowly burning cord with equally spaced knots tied in it. Each time a knot burned, this fact was tallied (digitally) and the subintervals were estimated by interpolation (analog). One more recent example is the odometer on all automobiles where the whole miles are given digitally while the tenths are estimated on a nonquantized analog-dial wheel. Another example appears in many digital clocks where the hours and minutes are given digitally on dials, but where the seconds are estimated on a continuous analog-scale wheel. Often in determining areas enclosed by curves, a combination of digital and analog methods are used. For example, the largest possible portion of the area may be estimated on a nonquantized analog-dial wheel. Once the area or areas are estimated by using a planimeter, an analog device.

There is an almost unlimited number of such examples that could be given to show that practical men have realized the importance of combined digital and analog computation, using the digital methods for greater accuracy and analog methods for simplicity and convenience. Today with elaborate analog and digital computers in widespread use it was again inevitable that practical men would seek to combine the salient features of each to produce a superior hybrid.

In previously presented papers, the following reasons were among those given for using only analog computing equipment for simulation:

1. Real time simulation is easy on an analog computer.
2. Actual analog hardware can be used in the loop.
3. An analog computer permits the engineer to get a "feel" for his problem by observing the results at the same time parameters are varied with potentiometers.
4. Analog equipment is comparatively inexpensive.
5. Analog computers require relatively short programming, debugging, and checkout time.

Similarly in the digital presentations, some of the reasons given for preferring only digital equipment for simulations were:

1. Digital computers are more accurate.
2. Digitally computed results are reproducible.

The strong features given for each type of computer are, in turn, the weak features of the opposite type. Thus, with each of the computer types having complementary strong points, it was only a matter of time before the interconnection of them was undertaken. It is natural to ask how it is possible to combine the advantages of analog and digital computers without combining their limitations; i.e., why is the resultant simulation not as slow as the digital computer and not inaccurate as the analog? The answer is, of course, the way in which they are combined. The analog equipment is used to simulate the high-frequency effects and noise while the digital equipment is used to simulate such things as relatively slow-speed navigational computers.

In the early part of 1954, under the direction of Dr. Walter H. Schwidetzky, Chief of Computers and Simulation at Convair-Astronautics, tentative specifications were written for an high-speed combined analog-digital and digital-analog converter. Among the more important considerations involved were the following:

1. The number of bits of digital information that should be converted.
2. The number of analog-digital channels and the number of digital-analog channels.
3. Conversion time for each channel.

After an evaluation was made, it was decided that a dynamic range of 100,000-to-1 would be more than adequate. To accomplish this, it was necessary to have 17 bits plus a sign bit where the most significant bit represented 50 volts and the least significant bit represented slightly less than 1 millivolt (mv) (actually $100 \times 2^{-17}$ volts).

For all work which could be contemplated, it was felt that 15 channels of analog to digital conversion and 10 channels of digital to analog conversion would suffice. Conversion time for each channel was limited by the state-of-the-art and the specified 100 microseconds per analog-digital (us per A-D) conversion and 25 us per D-A conversion have still been only approximately met.

It is interesting to note that originally the specified maximum sampling rate for any one channel of analog to digital conversion was from 1 to 20 per second and for digital to analog it was 1 to 100 per second; however, the equipment has successfully operated at analog-to-digital sampling rates of over 4,000 per second and at digital-to-analog sampling rates of over 5,000 per second. These speeds are limited by the 704 execution time and could be approximately doubled if a faster computer were available.

After all of the specifications were determined, the development and construction contract was let to Epsco, Inc., Boston in early 1955. While the 25-channel converter was being designed and built, combined simulations were performed using an Epsco Datrac single-channel analog-to-digital converter and single channel digital-to-analog converter to connect an 1103 computer with the analog computer on a trial basis. These tests were carried on successfully through the first half of 1957 proving the soundness of the technique. Tests were run on a simplified guided missile in-flight simulation and results were excellent.

Incidently, to investigate other uses

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