Combining Bundle Search with Buyer Coalition Formation in Electronic Markets: A Distributed Approach through Negotiation

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1. Introduction

A well-known example of building a purchasing strategy for buyers in electronic market is to form buyer coalitions (Buyer Club) [1] to enlarge the total quantity of goods purchased in each transaction. Buyers can obtain lower prices without buying more than their real demand. Another very interesting buyer strategy is called the “bundle search”, which addresses the situation where a buyer needs to buy different goods as a bundle. The problem is to find the optimal bundle that results in minimum cost [2]. It is very valuable to combine these two purchasing strategies for buyers to obtain greater discounts based on the different discount policies of multiple sellers.

There is little research work that considers both combinatorial coalition formation and bundle search together when the discount policies of sellers depend on the total cost of all goods sold in each transaction. One of the reasons can be that searching for the optimal buyer coalition structure and finding the optimal bundle are both computationally intractable problems. Finding the optimal buyer coalition structure can be translated into the weighted set packing problem [1], which is a NP-complete. Finding the optimal bundle is a NP-hard problem [2].

In this paper, we consider a purchasing problem in which a group of buyers are shopping from a certain group of sellers in a combinatorial market. The buyers have different shopping lists, and they are self-interested and geographically distributed. Different sellers offer different retail prices and different discount policies based on the total cost of all goods sold in one transaction. The problem is to find the optimal purchasing strategies that minimize the cost to buyers. Our approach is combining bundle search and buyer coalition formation through a distributed mechanism.

2. Problem Formalization

Formal Problem Definition: Let $G = \{g_0, g_1, \ldots, g_{l-1}\}$ denote the collection of goods items. There is a group of buyers $B = \{b_0, b_1, \ldots, b_{m-1}\}$, each of them has a shopping list denoted by vector $Q_i = (q_{i0}, q_{i1}, \ldots, q_{i,l-1})$, where $q_{ik}$ refers to the quantity of each item $g_k$ buyer $b_i$ needs to buy. There is a set of sellers $S = \{s_0, s_1, \ldots, s_{n-1}\}$ who can supply partial or all goods in $G$. Each seller $s_j$ has its own discount function $\delta_j(c): \mathbb{R}^+ \rightarrow \mathbb{R}^+$, which is the discount a buyer obtains when the cost of his purchase from seller $s_j$ is $c$. Also there is a retail price vector $P_j = (p_{j0}, p_{j1}, \ldots, p_{j,m-1})$ for each seller $s_j$. If seller $s_j$ has no good $g_k$ available, $p_{jk} = 0$. The objective of the problem is to minimize the cost to each buyer in $B$. To solve this problem and evaluate the performance, we need to define the following terms.

Discount Ratio: The discount ratio is defined as the ratio of the discount to the corresponding cost. Searching the maximum discount could be interpreted as finding the highest discount ratio the buyers can.

Coalition: A coalition (CL) is a subset of $B$.

Coalition Structure: A buyer can join multiple coalitions simultaneously by contributing some of his purchase items. A coalition structure is not a partition of buyers, but a partition of all goods that all buyers need to buy.

Coalition Value: The coalition value of a coalition $CL$ is defined as the sum of the savings that all members obtain through joining the coalition.

Value of Coalition Structure: The value of a coalition structure $CS$ is defined as the sum of the values of all coalition in the coalition structure.

3. Distributed Approach

The basic idea behind our approach is that a buyer makes his own decision based on maximizing his own utility. The coalition formation depends on negotiation among buyers instead of any mediation by a group...
leader. Hence, solving the purchasing problem turns into designing a mechanism that conducts each buyer’s decision to achieve both local and global optimality.

Our approach to solving the purchasing problem needs two steps. At first, buyers do their individual bundle searches to find the optimal bundle for their own shopping lists. If the discount ratios obtained from the sellers involved in the optimal bundle are the maximal discount ratios that the sellers can offer, buyers do not have to form or join any buyer coalition to increase the amount of discount they can gain. Otherwise, buyers start the second step of searching for coalitions proposed by other buyers or proposing new coalitions to related buyers.

We have developed a heuristic algorithm to solve the bundle search problem. It is called Maximal Gain Bundle Search (MGBS) algorithm of running time \(O(CNM)\), which is very efficient compared with the optimal algorithm of running time \(O(N^M)\) [2].

For the second step, we develop a distributed coalition formation mechanism based on multi-party explicit negotiation among buyers (Denoted by DCF-EN mechanism). There are three main issues in DCF-EN mechanism: the space of possible deals, the negotiation process, and the negotiation strategy.

**Space of Possible Coalitions:** A buyer only considers possible coalitions that include himself. The negotiation space is \(2^m - 1\) for each agent.

**Negotiation Process.** Buyers negotiate with each other by sending messages. The negotiation process for an agent handles every message from other buyers appropriately. Buyers’ decisions during a negotiation process are made based on their negotiation strategies.

**Negotiation Strategy.** In our current DCF-EN mechanism, all buyers use the same negotiation strategies as follows: Each buyer can propose multiple coalitions without waiting for the confirmation of the coalitions that have been sent out; Each buyer greedily accepts the best coalition that he can find; Each buyer who receives multiple coalition proposals can only accept one proposal at any given time; Buyers terminate their negotiation process when coalitions are formed (accepted), failed (refused) or time out; All buyers use a global clock.

**Claim:** The coalitions formed through DCF-EN mechanism are stable in the core [1].

**Proof:** Each buyer tries to join the best coalitions that he can find in our approach. The best coalition to a buyer is the one that maximizes his own saving. Any coalition that has been accepted by all its members must be the best coalition for all members that they can find. The values of subset coalitions of this coalition cannot be better than its value.

4. Simulation Results

![Figure 2: The Average Cost of Each Buyer](image)

![Figure 3: The Messages From One Buyer](image)

Figure 2 shows the average cost to each buyer with different purchase strategies (\(m\) refers to \(m\) types of goods that can be provided by \(n\) different sellers). Since the computational cost is too high to run an optimal algorithm for the purchasing problem, to compare our results with the optimal results for a certain purchase problem, we use the lower bound of the optimal cost for a buyer, which is the sum of the minimal retail prices of all his goods with obtaining the highest discount ratio in the market.

The results show combining bundle search strategy and buyer coalition formation strategy can reduce the cost more than just doing a bundle search and the cost is very close to the optimal cost. The average cost of each buyer is not increasing or decreasing significantly along with the number of buyers increasing in the market. Figure 3 shows that the number of messages from one buyer to another buyer are not increasing but decreasing sometime with the number of buyers increasing. For each buyer, the negotiation processes terminate appropriately without a global controller in the experiments.

5. References
