Revenue Maximising Agendas for Sequential English Auctions

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Abstract

This paper analyzes the way an auctioneer can arrange a series of sequential English auctions to maximize the total revenue. We consider the case in which heterogeneous common value objects are sold in these auctions, and each bidder has a budget constraint that allows it to buy at most one object. In such a setting, bidders need to determine how much to bid in each auction. Furthermore, bidders’ strategies change if the agenda (i.e., the order in which the objects are auctioned) changes. Consequently, the total revenue from the series of auctions can be changed by changing the agenda. Given this, the auctioneer clearly wants to determine the optimal agenda (i.e., the one that maximizes the total revenue). To this end, we first determine the equilibrium bidding strategies for players that know their own budget constraint, but have incomplete information about the others’ constraints. On the basis of these strategies, we determine the total revenue for different agendas.

1. Introduction

Multiple objects can be auctioned in two main ways: by using combinatorial auctions or by auctioning each object independently in a series of auctions (which can be arranged to occur simultaneously or which can be ordered sequentially). In this paper we focus on the latter approach and, in particular, on the sequential case.

In this work we specifically consider the case where the bidders have budget constraints (i.e., a bidder’s budget does not allow it to buy more than one object). Thus, a bidder needs to determine how much to bid in each auction to maximize its payoff across the complete set of auctions. For example, in sequential auctions for oil exploration rights, the price an oil company will pay for a particular area is affected not only by the area that is available in the current round, but also by the areas that will become available in subsequent rounds of leasing. Thus, it would be foolish for a bidder to spend all the money set aside for exploration on the first round of leasing, if potentially even more favourable sites are likely to be auctioned off subsequently. In other words, a bidder must determine bidding strategies that specify how much to bid in each individual auction. This decision making, and consequently the equilibrium outcome (i.e., the total revenue from all the auctions), depends on the auction agenda (i.e., the order in which the auctions are conducted). Given this, the auctioneer wants to determine the optimal agenda (i.e., the one that maximizes total revenue) for conducting the auctions.

In more detail, we consider the case where multiple heterogeneous common value objects are sold in a series of English auctions, one for each object. Furthermore, each bidder can buy at most one object. We first determine the equilibrium strategies and, on the basis of these strategies, we determine the revenue for each possible agenda.

Most of the existing work in sequential auctions has focused on multiple identical objects [5, 3]. The case of heterogeneous private-value objects has been studied in [4], and common-value objects are considered in [1], but both make the complete information assumption. Elmaghraby [2] studies sequential auctions for heterogeneous private-value objects, using second-price sealed-bid rules, which complements our focus on common-value case. Against this background, this work makes three important contributions to the state of the art in multi-object auctions. First, it analyses the bidding behaviour of budget-constrained agents for sequential auctions for common-value objects in an incomplete information setting. Second, it identifies the conditions under which it is possible for the auctioneer to determine the optimal agenda on the basis of the available information. Finally, it determines the seller’s optimal agenda.

2. Sequential auctions

There are two common-value objects $A$ and $B$ for sale and there are $n$ bidders $(s_1, s_2, \ldots, s_n)$. All players are risk neutral expected profit maximizers. The seller and all $n$ bid-
2.2. The seller’s optimal agenda

On the basis of the bidding strategies obtained in Section 2.1, we determine the total expected revenue for agendas AB and BA. The one that gives a higher revenue is the seller’s optimal agenda. Our study shows that, although it is not always possible for the seller to determine the optimal agenda on the basis of its available information, there exist some cases in which it can, and for these cases we determine the corresponding agenda. These results are summarized below.

1. If the budget constraints of bidders are independently and identically distributed over the interval [0, 1] according to the uniform distribution function, and \((a_1 - p_2(x)) > 1\) for all values of \(x\) in the interval [0, 1], the optimal agenda is to auction the object with the higher value first.

2. If the budget constraints of bidders are independently and identically distributed over the interval [0, 1] according to the uniform distribution function, and \((a_1 - p_2(x)) < 1\) for some values of \(x\) over the interval [0, 1], the optimal agenda depends on the actual bids and cannot be determined apriori.

Thus, in the first case it is possible for the seller to determine its optimal agenda on the basis of its information about the bidders’ budgets, while this is not so for the second case.

3. Conclusions and future work

This paper analyzes the strategies and outcomes of sequential auctions for budget-constrained bidders for multiple heterogeneous objects. We show that for such auctions the seller’s revenue can be changed by changing the agenda. We determine equilibrium bidding strategies for an incomplete information model in which the objects have common values. On the basis of these strategies we determine the optimal agenda. In future, we first aim to generalise our present analysis to \(n\) objects. We also want to consider other auction forms and determine whether the revenue depends on the auction form.

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References