Emergence of Control in a Large-Scale Society of Economic Physical Agents

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Abstract

We present a formal foundation for distributed large-scale control, unifying agent-oriented microeconomic market theory with the standard mathematical theory of control engineering.

1. Unification of Microeconomic and Control Theory

Large-scale industrial control in network settings is an example of real-world applications involving large numbers of interacting agents. Nonetheless, a fundamental theory of distributed intelligence underlying such applications is still lacking. This is the subject of the present paper.

Consider an interactive society of a large number of agents, each of which has an individual control task. What kind of control strategies will interactively emerge from this agent society, and how good are these with respect to both local and global control performance criteria? There are two, very different but both well-formalized, theories that can be brought to bear to this problem: control theory and microeconomic theory. The conceptual picture, then, is that agents are negotiating and trading with each other on a marketplace in order to acquire the resources that they need to achieve their individual control action goals, as indicated in Figure 1.

2. A Market Theorem for Control

We will specifically develop the microeconomic control theory for one significant class of local controllers called PID control. It is a standard tool to solve control problems [1] that is widespread in industrial control engineering. The general equation expressing the PID control strategy is

\[ r(t) = K^P x(t) + K^I \int_0^t dt' x(t') + K^D \frac{dx}{dt} x(t), \]  

(1)

Here, \( r(t) \) is the input resource variable, \( x(t) \) is the output state error variable, the difference between the actual state and the goal state (setpoint), and the \( K \) are called gain constants. For example, in climate control of large office buildings \( x \) stands for the error in room temperature and \( r \) is the cooling power.

Consider a large collection \( \alpha = 1, \ldots, N \) of independent local PID controllers (in practice, \( N \) may easily run in the hundreds or even thousands). Eq. (1) can be more concisely written in operator notation as

\[ r_\alpha = \mathcal{O}_\alpha x_\alpha, \]

where \( \mathcal{O}_\alpha \) is a linear operator operating on the local error state variable \( x_\alpha \). This PID control rule defines the agent’s local demand function without taking possible resource constraints into account. If, on the other hand, the total resource is constrained, it is traded on a marketplace that thus serves as a resource allocation mechanism.

Definition 2.1 Let each independent local PID controller be represented by an agent \( \alpha \), with its utility function defined by \( u_\alpha = f_\alpha(r_\alpha) \), where \( f_\alpha \) is a strictly concave function of \( r_\alpha \) that is twice continuously differentiable on a suitable interval, and that has its maximum at the local resource value \( r_\alpha = \mathcal{O}_\alpha x_\alpha \) as given by the PID control equation Eq. (1). Suppose further that the total available resource is
scarce, so that we have
\[ 0 \leq \sum_{\alpha=1}^{N} r_{\alpha} = R_{\text{max}} \leq \sum_{\alpha=1}^{N} O_{\alpha} x_{\alpha} \equiv R_{\text{unc}}. \tag{2} \]

Finally, let all agents be self-interested utility maximizers and let them be competitive (i.e. price takers).

**Theorem 2.1** Assuming the above definitions, the following statements hold:

A. There exists a resource allocation \( r^*_{\alpha} \) that is a global maximum to the optimization problem:
\[
\text{Max} \sum_{\alpha=1}^{N} f_{\alpha}(r_{\alpha}) \quad \text{subject to} \quad \sum_{\alpha=1}^{N} r_{\alpha} = R_{\text{max}}, \text{and this global maximum is unique.}
\]

B. The same resource allocation \( r^*_{\alpha} \) is identical to the competitive equilibrium of a market in which each individual agent maximizes its utility \( f_{\alpha}(r_{\alpha}) \) within its budget, whereby the market is clearing (i.e. \( \sum_{\alpha=1}^{N} r^*_{\alpha} = R_{\text{max}} \)) and \( p^* \) is the market clearing price.

C. The resource allocation \( r^*_{\alpha} \) obtained as outcome of this competitive equilibrium market is Pareto optimal.

The formal proof is given in a full paper available upon request from the authors. The optimization problem of statement A reflects how a central controller, that oversees all local control agents, will look at the situation in a hierarchical, top-down manner. In contrast, the market problem of statement B represents the situation through the eyes of the individual agents in a bottom-up and emergent fashion. Our market theorem then shows that there is an *outcome equivalence* between the two approaches. Moreover, statement C says that the resulting resource allocation \( r^*_{\alpha} \) is optimal both locally and globally, in other words, it is optimal for each agent individually as well as at the agent society level.

### 3. Real-World Microeconomic Control

Our approach has been designed such that it takes full advantage of existing established control theory, in contrast to other agent-based control approaches (such as Huberman’s and Lesser’s approaches [2, 3]). Microeconomic control is actually a special type of what is known in control engineering as *adaptive* control. It embodies an, online and self-organizing, adaptation of the local control strategies when overall resource limitations come into play.

Our theory allows for a wide range of practical applications. It subsumes the agent applications carried out especially under the heading of market-based control (see e.g. [4]). For a practical implementation of electronic markets for multi-agent control tasks, any suitable resource-oriented market protocol will do, such as the ones discussed by Ygge and Akkermans [5, 6]. This type of algorithm has been successfully used in industrial field experiments on agent-based comfort management in large buildings by means of our HOMEBOT agent concept (see e.g. [7]). Current investigations concern a variety of industrial applications of distributed intelligence in the area of ICT-based power network control, including real-time demand-supply matching, intelligent load shedding, and online demand-response strategies and services.

In sum, our theorem proves two important properties about agent-based microeconomic control: (1) computational economies with dynamic pricing mechanisms are able to handle scarce resources for control adaptively in ways that are optimal locally as well as globally (‘societally’); (2) in the absence of resource constraints the total system acts as collection of local independent controllers that behave in accordance with conventional control engineering theory. The latter theory does not deal with constraints on shared resources in large-scale control systems. This is the added value of the present microeconomic control approach. It provides the theoretical underpinnings for a wide range of industrial distributed large-scale control applications.

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**References**


