Editor: George Cybenko
Send announcements to kclark-fisher@computer.org

A LOOK AT LATTICE BOLTZMANN EQUATIONS

By Bruce M. Boghosian

The Lattice Boltzmann Equation: For Fluid Dynamics and Beyond, by Sauro Succi
Oxford University Press, New York, 2001, ISBN: 0198503989, US$100

DURING THE PAST 10 YEARS, A NEW CLASS OF ALGORITHMS BASED ON THE LATTICE BOLTZMANN EQUATION (LBE) HAS BEEN DEVELOPED FOR COMPUTATIONAL FLUID DYNAMICS (CFD). THESE NOVEL AND FASCINATING ALGORITHMS WERE INSPIRED BY kinetic theory—a branch of statistical physics. They defy the conventional wisdom of CFD in that they provide stable, fully explicit differencing schemes, with no need for elliptic solvers or upwind differencing. Yet, they are also remarkably simple. In fact, the typical reaction of long-time CFD practitioners when encountering lattice Boltzmann algorithms for the first time is often something like, "That can't possibly work. It's too easy."

Sauro Succi beautifully describes the first 10 years of research on these remarkable algorithms in *The Lattice Boltzmann Equation: For Fluid Dynamics and Beyond* (Oxford University Press, 2001). The author is a principal figure in the algorithms' discovery and development, and his style is clear, complete, and entertaining. In addition to presenting the method's general theory and several of its variants, he describes its history in detail. In particular, he chronicles the series of discoveries that led researchers to understand that the key to discretizing the Navier-Stokes equations, which describe the dynamics of a viscous fluid, was to instead discretize the LBE, from which they can be derived.

A Short History of LBE

The Boltzmann equation is remarkably resilient to drastic simplification and idealization of its microscopic velocity degrees of freedom; for example, if particles' velocities are restricted to a finite, discrete set of possibilities, the Boltzmann equation might still make sense. Because these are exactly the degrees of freedom that are integrated away in the reduction to a hydrodynamic description, the Navier-Stokes equations can emerge intact. The LBE is then nothing more than the Boltzmann equation for particles hopping on a lattice with velocities restricted to the lattice vectors. Kinetic theoretical tools can be unleashed to show that the system's bulk behavior is described by the Navier-Stokes equations, which are thus emergent.

At first glance, it is difficult to believe that a lattice model could capture all the intricacies of fluid motion. After all, the discrete nature of a spatial grid and the preferred directions of its lattice vectors seem to stand in stark contrast to the smooth and isotropic nature of fluid dynamics. Indeed, as the author describes, faithful and accurate lattice models of fluids came as rather a surprise when they were first introduced in the mid 1980s. The first such models were called *lattice gases* and consisted of discrete particles moving on a regular spatial grid. As researchers investigated these particles' single-particle distribution function, they developed varieties of progressively simpler lattice Boltzmann models. Because these models evolved distributions in time rather than discrete particles, they lacked the statistical noise that plagued the measurement of hydrodynamic quantities in lattice-gas simulations. Thus, by the early 1990s, these replaced lattice gases as the most successful discrete-velocity algorithms for CFD.

The physical motivation for lattice models was simple: many fluids that differ dramatically at the molecular level—such as air, water, and molasses—have bulk behavior that is well described by the Navier-Stokes equations of viscous hydrodynamics. Only the viscosity—a parameter in the equations—differs from one fluid to another. This is despite the fact that the only common feature these materials share is that their molecular collisions conserve mass and momentum. Ultimately, this observation owes to the deeper fact that the Navier-Stokes equations are a dynamic renormalization group fixed point of molecular models with a conserved mass and mo-

 Copublished by the IEEE CS and the AIP 1521-9615/03/$17.00 © 2003 IEEE

mentum. Kinetic theory tells us that the molecular-level details do not matter; the Navier-Stokes equations will be the generic dynamical equations describing the bulk motion. The inventors of the first lattice-gas models turned this observation on its head. Given that a wide class of particulate systems with conserved mass and momentum will have bulk behavior obeying the Navier-Stokes equations, they sought the simplest such dynamics possible. Simulate that, they reasoned, and you would be simulating a fluid.

Practical Issues

In spite of their very different motivations, lattice Boltzmann algorithms bear strong algorithmic resemblance to explicit finite-difference algorithms, which means that they are easy to parallelize. Succi devotes several sections in his book to practical issues surrounding the computer simulation of these algorithms, and he includes a pointer to sample lattice Boltzmann code for a Bhatnager-Gross-Krook collision operator. He devotes a section to describing the lattice Boltzmann model's relationship to other CFD algorithms, including fully Lagrangian schemes. Succi is candid about the algorithm's strengths and weaknesses, and in a useful section titled "Who Needs LBE?" he categorizes users into "must use," "should use," "can use," and "don't use!" classes. The discussion is at once entertaining and useful.

In the "Beyond Fluid Dynamics" chapter, the author takes us on a tour of a zoo of variants of the basic lattice Boltzmann algorithm that researchers have concocted over the past decade for a variety of different applications. He describes model extensions for compressible, thermohydrodynamic viscous flow. Succi describes lattice Boltzmann models for, inter alia, immiscible fluids, amphiphilic fluids, colloids, and polymers. In situations in which competing models exist, he describes each and lists their strengths and weaknesses. Remarkably, he also describes lattice Boltzmann model extensions for the simulation of the Schrödinger and Dirac equations.

Even though specialists and practitioners will use this text as a reference book for some years to come, picking it up and reading it from cover to cover is remarkably easy. Succi wrote it to be accessible to graduate students, and even advanced undergraduates will benefit from it. Its readers will surely include engineers, physicists, and algorithm developers, and the author has taken care to make the book accessible to readers from these backgrounds. The book includes exercises at the end of each section, making it useful perhaps as a supplementary text in a course on CFD.

Discrete kinetic models of fluids is a fast-paced field, and *The Lattice Boltzmann Equation* covers developments through 2001. Although future editions are sure to be thicker—there are already some remarkable improvements in entropic and multiple-time-scale relaxation lattice Boltzmann collision operators just published in 2002—the present work is likely to stand as a seminal publication in this exciting area of research. CiSE

Bruce Boghosian is a professor of mathematics at Tufts University. His research interests are theoretical and computational fluid dynamics, turbulence, quantum computation, and knot theory. He has a BS in physics and an MS in nuclear engineering from MIT and a PhD in applied science from the University of California, Davis. Contact him at bruce.boghosian@tufts.edu.