IT ALL COMES DOWN TO $B_{NEW} \Leftarrow B_0 \oplus (B_1 \vee B_2)$

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THE FORMULA IN THE TITLE IS THE BINARY VERSION OF RULE 30 FROM STEPHEN WOLFRAM'S NEW BOOK, *A NEW KIND OF SCIENCE* (WOLFRAM MEDIA, 2002). RULE 30 IS A CELLULAR AUTOMATON THAT LOOKS AT THREE BITS AND GENERATES A NEW ONE, B_{NEW}, FROM THOSE THREE. IT IS SAID TO GENERATE patterns that are complex and indistinguishable from random ones, a remarkable result for such a simple algorithm. As of this writing, Amazon.com ranks Wolfram's book number 18 in sales of all books. To put this in perspective, I checked the rank of the Feynman lectures (Addison-Wesley, 1970), which was 2,003, and of Marc Kac's little masterpiece, *Statistical Independence in Probability Analysis and Number Theory* (Mathematical Assoc. of America, 1959), which was 1,040,441. Even if the sales figures were less impressive, a scientific book whose central premise is that the universe is an algorithm is bound to be of great interest to anyone concerned with computational science and engineering, and many other people as well. This issue of *CiSE* contains two reviews of *A New Kind of Science*. My purpose here is to make a few remarks about the directions this new science might take.

Although the book is beautifully produced and rather inexpensive for such a high-quality production, these facts alone don't explain its popularity. Its appeal, I think, is due to the breadth of vision and the magnitude of the claims it makes. I'm thrilled by the vision but also slightly worried by the claims.

Suggestion 1: Make it a science

Science is important because of its ability to predict. Ideally, the predictions are quantitative, precise, and verifiable via experimental observation. We can predict, for example, that if we drop two objects from a height of two meters in a room that contains nothing else (including air), the objects will reach the floor in 0.783 seconds and that when they do, their velocities will both be 7.67 meters per second. We can check this prediction with a clock and a measuring rod. At present, I don't see how to use cellular automata to make predictions about the outcome of physical experiments.

Suggestion 2: Prove things

The proof's purpose is to decrease the risk of reaching a false conclusion due to an error in reasoning. Naturally, not every important scientific fact is amenable to proof. For example, it's likely that we can't actually give a rigorous proof of the second law of thermodynamics, although Marc Kac comes close in the last chapter of the book mentioned earlier. Although we can't prove some of the most important things, proving what we can prove is essential to the health of an analytical and computational science. For one thing, it forces us to make careful definitions. A careful definition of complexity might help clarify the claims made for Rule 30.

Suggestion 3: Connect the new science with the old

A paradoxical feature of probability theory, when applied to statements about real numbers, is that things that happen with probability one seldom occur in "real life." A related phenomenon is that some things appearing to be extremely special are actually common and even universal. The standard joke goes as follows (**P** is a probabilist, and **G** is just some guy giving a reasonable response):

P: Pick a number between zero and 10.
G: Four.
P: How interesting that you picked an integer! That's a zero-probability event.

Although nobody would actually have such a conversation (except perhaps in the lounge of your local mathematics department), some real-world probability facts are just as strange. With probability one, every real number is irrational—in fact, transcendental—but not many transcendental numbers are known explicitly. Stranger still, with probability one, all real numbers are "normal," meaning they are provably complicated. Only a handful of normal numbers are known explicitly.

Deeper results reveal more surprises. If we choose a number x between zero and one uniformly at random and write out the digits of the continued fraction expansion for x, say $a_1, a_2, \ldots, a_k, \ldots$, then the average of the $\log(a_k)$ converges to a unique constant K for all x, except for perversely chosen zero-probability cases. In other words, the constant K is universal. The proof is an application of the ergodic theorem (again, see Kac's book).

Is this result any less amazing or fundamental than Wolfram's Principle of Computational Equivalence? I think not. For all we know, the Principle of Computational Equivalence could be a corollary of this result. In any case, it is science, it is proved, and it is firmly connected to the rest of science. Cellular automata might very well turn out to be the foundation of a new, more vigorous science. There certainly is something absolutely fundamental about algorithms. But much rigorous work remains to be done before we can build an edifice on this new foundation.

Correction

David Beazley received his PhD from the University of Utah, not from the University of Oregon (as the last issue claims). We apologize for any confusion this error might have caused.