A Numerical Method for Performance Analysis of ATM Multiplexers

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Abstract

This paper presents a numerical method for performance analysis of multiplexers in a wide variety of environments. Using this method, performance analysis of ATM multiplexers fed with two important classes of arrival processes, namely correlated on-off and periodic traffic, has been accomplished. Unlike generating function approaches, the proposed analysis method is in time domain and is appropriate for obtaining dynamic measures of ATM multiplexers. In this paper numerical results are found and displayed to show the effect of high priority periodic traffic on performance measures of low priority correlated streams.

1: Introduction

One of the major attributes of the broadband networks is the statistical multiplexing capability of different traffic types, that results in an efficient utilization of network resources. An important issue in the design of ATM networks is finding accurate numerical methods for finding appropriate buffer sizes at the multiplexer nodes to avoid excessive losses during all cell slots. In [1] a fluid flow approximation has been used to investigate the performance measures of multiplexers fed with bursty traffic. There it has been observed that asymptotic behavior of buffer content is geometrical. In [2] the aggregate traffic of ON_OFF sources is approximated by means of a two-state Markov modulated Poisson process (MMPP). In [3] and [4] using spectral decomposition method, an asymptotic analysis for derivation of performance measures of multiplexers fed with discrete heterogeneous on-off models has been devised. Unfortunately the above mentioned approaches are not well suited for the study of dynamic behavior of the multiplexers. Despite the applicability of the numerical approach proposed by Neuts [5], the number of involved equations in the study of dynamic performance measures grows quickly and might not be a trivial task.

Here our goal is to show a simple and accurate numerical method for performance analysis of multiplexers fed with correlated on-off traffic alone or as a mixture with deterministic (periodic) traffic in a discrete environment under transient or stationary conditions. The basis of this algorithm is briefly an iterative calculation of buffer occupancy at each cell slot. The proposed method in here is conceptually similar to the original method devised by Jenq, [6]. The proposed numerical method unlike most of the available analysis approaches, does not rely on the frequency domain transformation of the buffer occupancy. The algorithm is first explained for the case where the multiplexer node is loaded with correlated on-off traffic sources alone, based on which cell loss rising time of the multiplexer has been obtained. Later the method is extended to consider the effect of periodic traffic on the performance of correlated traffic.

Using OPNET package, a simulation model has been build, and its results have been compared with those obtained by the proposed numerical method. Simulation and numerical results are shown to be close in all the accomplished experiments.

The organisation of the paper is as follows. Section 2 presents the discrete traffic source model. In section 3 the numerical method for performance analysis of correlated and periodic traffic are explained. Section 4 gives a brief description of the simulation model.

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In section 5 the quantitative results of the paper are presented, and finally in section 6 conclusion is made.

2: Traffic Model

In the following investigations binary Markov traffic model with burst and silence states have been used. In this traffic model, cells are only generated during the burst state. To generate a correlated output traffic, burst and silence duration lengths are modelled as two geometrically distributed random variables, with different average values. Time is slotted according to the peak-cell-rate of the source nodes, and slot is a fixed-length time interval. In analysing buffer occupancy of the multiplexer, it is necessary to calculate transition probability matrix of the aggregate traffic. Assuming N homogeneous on-off traffic sources feeding the multiplexer, aggregate traffic will have $N+1$ possible states, for which a transition probability matrix having $(N+1)$ by $(N+1)$ elements are computed. The elements of this matrix (T), are obtained using (1).

$$T(i+1, j+1) = \sum_{r=\max(0,(j-i))}^{\min(j,(N-i))} \binom{N-i}{r} a^{(N-i-r)} (1-a)^{r} \times \binom{i}{i-j+r} b^{(j-r)} (1-b)^{(i-j+r)} \quad i,j \in (0,N) \qquad (1)$$

Here $T(1,1)$ is the probability of remaining in the idle state with no (zero) active source, and $T(i,j)$ is the transition probability from $(i-1)$ to $(j-1)$ active source state. In the above equation 'a' and 'b' are ON-OFF traffic model parameters as shown in Fig1.

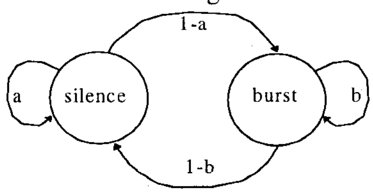

Fig. 1. Burst silence traffic model

Utilisation factor for this source model is equal to $(1-a)/(2-a-b)$, and the average burst and silence length are respectively $1/(1-b)$, and $1/(1-a)$. Obviously the sum of elements in each row of T will be equal to one. Further, successive multiplication of T with itself gives the distribution for the number of active sources, which can be obtained by direct binomial method for on-off traffic sources as follows:

$$pr\{S = S_i\} = \binom{N}{i} \eta^{i} (1-\eta)^{N-i} ;$$

where η is equal to the utilisation factor of each traffic source

3: Analyses

3.1: Numerical Method

In describing the numerical method, the model shown in Fig(2) has been used.

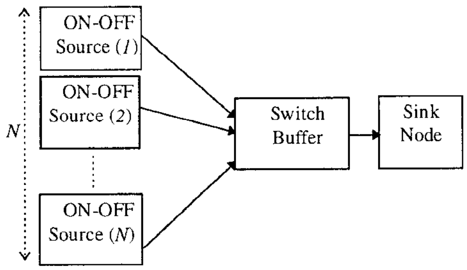

Fig 2. output buffer switch

The numerical method is based on using buffer occupancy matrixes q_s, q_a, which are buffer occupancies after service completion, and after arrival instants respectively. Each element of q (q_s or q_a) , gives the probability of a queue size versus a state of the traffic source. This matrix is obviously different from the one step queue length transition matrix.

The row index of q, is assumed to be equal to the number of available cells in the buffer, and is limited to the maximum buffer size of the multiplexer. Column index of q gives the number of active sources and is limited to one more than the number of connected streams N. Further assuming $q(1,1)$ to be the probability of empty queue and no active source, $q(i,j)$ will be the probability of i-1 cells in buffer, and j-1 sources, active. The row and column sizes of q matrix, will be equal to $(B+1)$ and $(N+1)$ respectively, where B is assumed to be the buffer capacity in cell units and N is the number of connected streams. Further properties of this matrix are shown in eq(2):

$$pr\{n = i\} = \sum_{j=1}^{N+1} q(i+1, j); \qquad i \in (0 \text{ to } B) \qquad (2a)$$

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$$pr\{s=j\} = \sum_{i=1}^{B+1} q(i,j+1); \quad j\in(0 \ to \ N) \tag{2b}$$

$$\sum_{i=1}^{B+1}\sum_{j=1}^{N+1} q(i,j) = 1 \tag{2c}$$

$$q(i,j) = 0 \qquad i,j\leq 0 \tag{2d}$$

In above, n is the queue length, s is the state of the aggregate traffic, N is the number of connected sources, and q represents both q_s and q_a. It should be emphasised that for a binary Markov traffic model, a cell is emitted in each slot, while in the active state, and no cell is generated during a silence mode. As a result, when the number of active sources is changed to j, j cells are expected to arrive in that slot. Based on the previous assumptions, and assuming service rate to be equal to the peak cell rate of the traffic sources, then the proposed numerical algorithm can be described as follows:

1)-Calculation of Transition probability matrix T: Transition matrix is obtained using eq(1).

2)-Iteration stage: In this stage, probability distribution of buffer occupancy q in each slot is calculated through the following steps:

2.1)-Derivation of the buffer occupancy matrix right after arrival q_a using eq(3a). This step is justified since we have:

$$pr\{n_{a,r} = i \cap s_r = j\} = \sum_{s_{r-1}=0}^{N} pr\{s_r = j / s_{r-1} = k\}.$$

$$pr\{n_{s,r-1} = i-j / s_{r-1} = k\}$$

Where $n_{a,r}$ $n_{s,r}$, s_r, are respectively queue length after arrival, queue length after service completion instant, and the state of the aggregate traffic in slot r, N is the number of streams connected to the multiplexer, pr{s_r=j |s_{r-1} =k) is the transition probability from state k to j, and finally pr{$n_{s,r-1}$ = i-j | s_{r-1} = k) is the probability of having i-j cells in the buffer after a departure and while in state k. Eq(3) shown below is based on the above relationship.

$$q_{a,r}(i+j-1,j) = \sum_{k=1}^{N+1} q_{s,r-1}(i,k)\times T(k,j)$$

$$\text{for} \qquad i\in(1,B+1) \tag{3}$$

2.2) Derivation of buffer occupancy matrix after service completion, q_s, using eq(4a): At the end of a complete cell transfer, it follows that;

$$pr\{(q_{s,r} = i-1)\cap(s=s_j)\} = pr\{(q_{a,r} = i)\cap(s=s_j)\},$$

where it is assumed that the cell transmission is completed right before the cell slot boundary, and cell arrival to the multiplexer buffer occurs right after the boundary. Therefore buffer occupancy matrix after service completion instant, q_s, can be found as shown below in (4).

$$q_{s,r}(i,j) = q_{a,r}(i+R,j), \ i\in(2,B), j\in(1,N+1) \tag{4a}$$

$$q_{s,r}(1,j) = \sum_{i=1}^{R+1} q_{a,r}(i,j), \quad j\in(1,N+1) \tag{4b}$$

$$q_{s,r}(B+1,j) = \sum_{i=B+1+R}^{N+R+1} q_{a,r}(i,j) \quad j\in(1,N+1) \tag{4c}$$

where R is the ratio of link capacity to PCR. It should be mentioned, that the above iteration steps has to be repeated until convergence towards the stationary state is achieved. According to (5), convergence is assumed to be complete when the changing rate of elements of the buffer occupancy matrix are less than a threshold value ε, eg 10^{-9}.

$$\frac{\sum_{j=1}^{N+1} q_{s,r-1}(1,j)}{\sum_{j=1}^{N+1} q_{s,r}(1,j)} - 1 < \varepsilon \tag{5}$$

where a downward convergence is assumed, since $q(1,1)$ is set equal to one at the beginning.

3)Calculation of performance measure: This stage is a simple averaging procedure using the obtained buffer occupancy.

3.2: Periodic Traffic

In obtaining the effect of periodic traffic on the performance measures of multiplexer, analytical methods cannot be easily used. As shown in the following, performance measures can be easily obtained on the basis of the above numerical method.

This periodic behavior is especially probable for ATM multiplexers near to the end terminals where the cells emanating from the same traffic source are not interleaved with the cells of a large number of streams. Here it is assumed that at the end of each period, a

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deterministic number of cells arrive to the multiplexer. Cell arrival of periodic traffic can happen in two different ways i.e. batch or back-to-back type of arrival. Assuming that periodic cells have the highest priority, then CLR at the beginning of each period increases sharply for lower priority connections. Transmission of the non-periodic cells resume after a complete transfer of the higher priority cells. Eq(6) shows the effect of periodic batches of length M on buffer occupancy, right after the arrival of periodic batches.

$$q_a(i+M, j) = q_a(i, j), \text{ for } j \in (1, N+1) \qquad (6a)$$

$$q_a(i, j) = 0, i \in (1, M), \qquad j \in (1, N+1) \qquad (6b)$$

Where q_a on the right side of eq(6b) is the buffer occupancy matrix right after arrival of non_periodic traffic, and q_a on the left side of (6a, 6b) is the occupancy matrix after arrival of both periodic and non_periodic traffic. For back-to-back arrival the above relation has to be repeated for the duration of the burst.

Under stationary conditions, each slot of the period has a different average CLR. In this case maximum CLR might be much higher than the average CLR of a complete period. This might result in a major reduction of quality of service (QOS), at the periodic arrival instants.

Although we have restricted our calculations to constant size batches, other batch size distributions could also be handled. For a periodic random batch size, shift operation has to be replaced with convolution, ie. at the end of each period, columns of the occupancy matrix must be convolved with the batch size distribution vector (at batch arrival cell slot).

4: Simulation Model

The simulation model is similar to that shown in Fig(2). As before burst and silence duration of the traffic sources, are two independent, geometrically distributed, random variables, with different average values. Measured averages of the utilization factor, and first and second order moments of the burst and silence periods of the traffic sources are found to be equal to the theoretical values. The displayed simulation results have a tolerance range of $\pm 10\%$ within a 95 percent confidence interval and therefore reflect accurate references.

Average queue size and CLR of the multiplexer have been chosen as a comparative measure between simulation and numerical results. Queue length has been sampled at constant time intervals during the simulation. It should also be mentioned that queue size does not include the cell being serviced (transmitted).

5: Numerical Results

The following results are obtained for a 100 cell multiplexer buffer, with 10 input links, (N=10). For multiplexers fed with binary Markov traffic sources alone, similar results are obtained by the proposed numerical method and the generating function approach [3]. Fig(3) shows this similarity of the results. Additionally almost all of the analytical results lie within the 95% confidence intervals of the simulation results.

An interesting feature of the proposed numerical method is its capability in finding the performance measures of the multiplexer during its transient states, see Fig(4). In this figure rise time is the minimum duration after which cell loss probability of a congested node rises above a specific loss threshold. In Fig(5) the effect of link capacity-to-*PCR* ratio, on the cell loss probability of a multiplexer is displayed. Exactness of the numerical approach is further observed by a comparison of the simulation and numerical results.

In Fig(6) and Fig(7), average queue length and average CLR versus average on period for different utilization factors are displayed. As expected both the simulation and numerical results show an increase of average queue size and CLR with respect to a corresponding increase in utilization and "average on" period.

In order to show the effect of periodic traffic on performance measures of the multiplexers, results in figures (8) and (9) are obtained for a mixture of binary Markov and periodic traffic sources. Fig(8) shows maximum CLR for different periodic batch sizes. It has been observed that CLR has its maximum value, at the arrival instants of the periodic cells. In these experiments

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average utilization of the periodic traffic source has been kept constant by keeping a constant ratio between the batch size and the period length. As expected maximum CLR, increases for larger batch sizes (and larger periods). Obviously arrival probability of large periodic batches increases for higher utilization factors of periodic traffic load, which should be avoided using proper scheduling strategies, [7]. It is worth mentioning that despite a rapid increase of the maximum CLR versus batch size, average CLR increases only by a small fraction, which shows that average CLR cannot always be a good measure of QoS. In Fig(9) CLR at different slots within a period are displayed for two cases. As it can be seen the maximum achieved CLR for periodic batch arrivals is greater than the same measure in the case of consecutive cell arrivals, which is intuitively correct. By spreading the consecutive cells, maximum cell loss can be further reduced.

6: Conclusion

In this paper a numerical method for the study of performance measures of multiplexers fed with correlated and periodic traffic under dynamic and stationary states has been described. Sensitivity towards different traffic parameters like "average on" period, and "utilization" has been shown. It has been found that periodic batch arrivals increase CLR at arrival instants, which despite totally low average values of CLR, may result in a poor quality of service for low priority bursty traffic. As an example, with a specific utilization (0.1) of periodic traffic, maximum CLR for a consecutive arrival of the periodic cells is equal to one third of the CLR for batch arrival of the periodic cells Further it is found that in comparison to periodic batch arrival a consecutive entrance of periodic cells results in a comparatively better performance which can be further improved by selection of an appropriate cell scheduling policy of the periodic cells.

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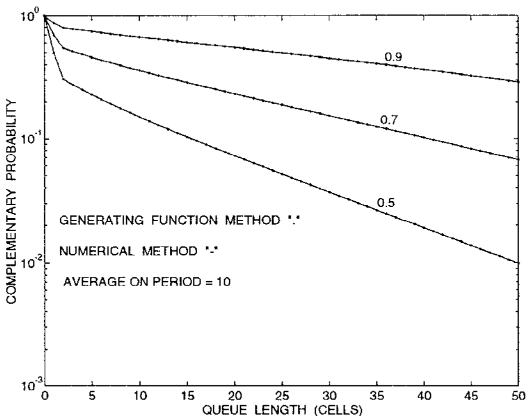

Fig. 3. Queue length tail distribution pr($q \geq i$) for different total average utilization, generating function method: '.' numerical method: '-' N=10, (η = **0.5**, η = **0.7**, η = **0.9**)

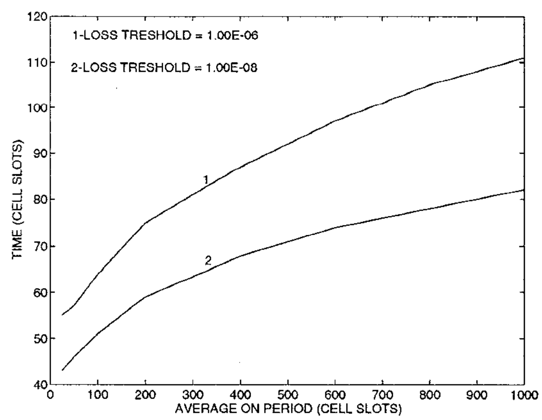

Fig. 4. Minimum rising time versus T_{on} for different loss thresholds **1**- 10^{-6}, **2**- 10^{-8} ($N.\eta$ = 0.8, N=20, buffer size = 100 cells)

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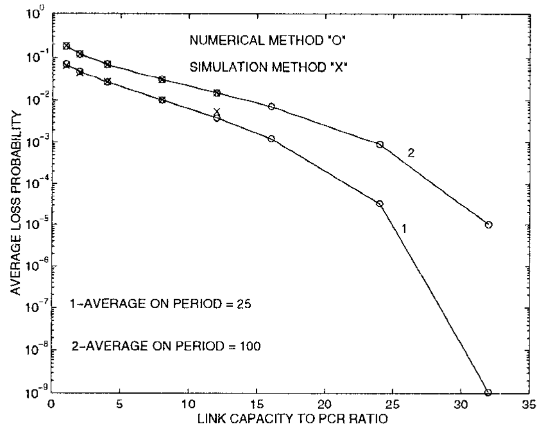

Fig. 5. CLR versus link capacity ($N.\eta.PCR/C$) = 0.8, N = 40, B= 100 cells, **1-** T_{on} = 25, **2-** T_{on} =100

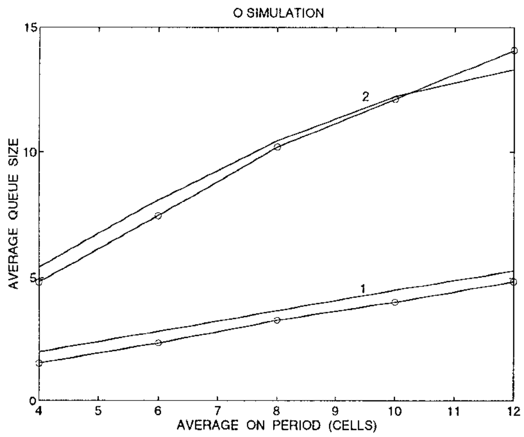

Fig. 6. Average queue size versus average on period for different total utilizations (**1-**η = 0.5, **2-**η = 0.7)

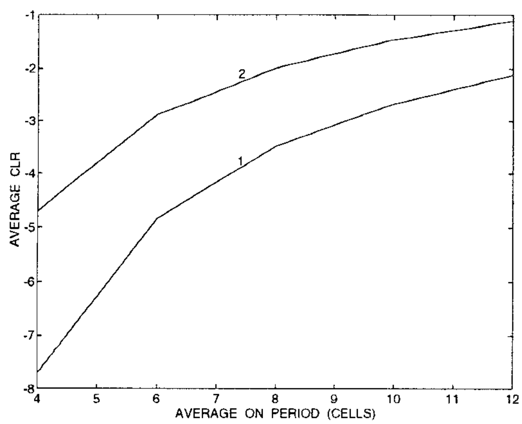

Fig. 7. Cell loss ratio versus average on period for different total utilization factors, N =10, (**1-** η = 0.5, **2-**η = 0.7)

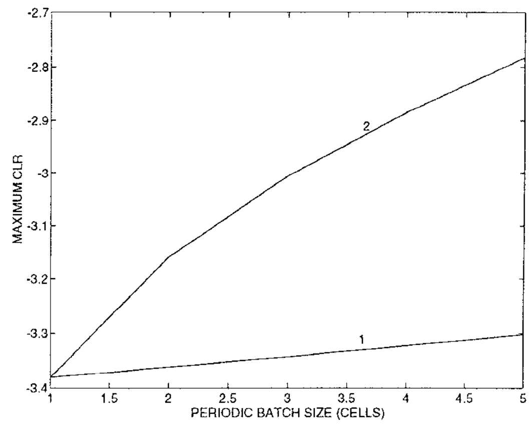

Fig.8. Maximum CLR versus different periodic batch sizes (periodic traffic utilization = 0.1, total utilization of bursty traffic = 0.48, N=8, average duration of active period = 10 cells) **1-** average CLR, **2-** maximum CLR

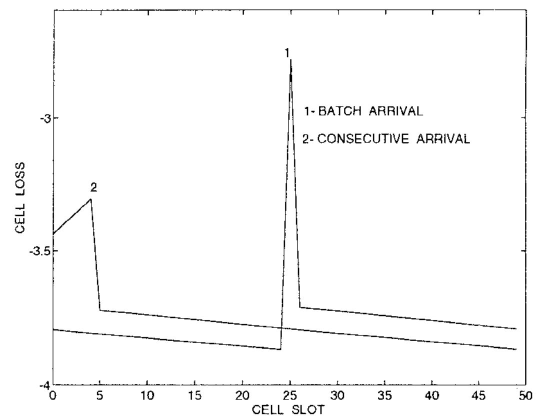

Fig. 9. Stationary CLR versus different cell slot of a period (batch size = 5, batch interarrival = 50 cells, periodic traffic utilization = 0.1, total utilization of bursty traffic = 0.48, N =8, average duration of the active period = 10 cells)

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