

Hierarchical Statistical Models for the Fusion of Multiresolution Image Data

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Abstract

This paper presents a class of non-linear hierarchical algorithms for the fusion of multiresolution image data in low-level vision. The approach combines non-linear causal Markov models defined on hierarchical graph structures, with standard bayesian estimation theory. Two random processes defined on simple hierarchical graphs (quadtrees or "ternary graphs") are introduced to represent the multiresolution observations at hand and the hidden labels to be estimated. An optimal algorithm (inspired from the Viterbi algorithm) is developed to compute the bayesian estimates on the hierarchical graph structures. Estimates are obtained within two passes on the graph structure. This algorithm is non-iterative and yield a per pixel computational complexity which is independent of image size. This approach is compared to the multiscale algorithm proposed by Bouman et al. in [3] for single-resolution image segmentation (that we have extended for multiresolution data fusion).

1 Introduction

In a growing number of applications in computer vision and low-level image processing, one has to process and combine image data stemming from multiple sensors. Multisource or multispectral data fusion is indeed known to cope efficiently with the ill-posed nature of most low-level vision problems. Since multisource data are generally available at different resolutions, one has to develop adequate models and algorithms to obtain a consistent data fusion.

In this paper, we describe a statistical framework, relying on Markov models defined on hierarchical graph structures, for the fusion of multiresolution image data. Causal Markov models [3, 9] have been adopted to represent multiresolution image data as well as the label fields corresponding to the result of the fusion process. Optimal *non-iterative* algorithms are derived on the hierarchical graph structures. They yield optimal bayesian image data fusion, with a per-pixel computational complexity which does not depend on the image size.

Let us notice that the combination of Markovian models and multiresolution algorithms in a consistent and tractable mathematical framework is an intricate the-

oretical and practical issue. The first approaches proposed in the literature were essentially based on heuristic associations of Markov models with multiresolution representations of the images to process [2, 7, 10]. Gidas, [4] has described a consistent mathematical framework for multiscale Markov modeling, based on the *renormalization group* approach. Unfortunately, this standard technique in statistical physics does not lead to tractable computational schemes (apart from particular models and scale transformations [4]) because of the loss of locality of the model at the coarse scales. Jeng [6] and Pérez *et al.* [12] have recently studied the loss of locality in a periodic subsampling of Markov models. Alternate approaches, involving various kind of coarsening operators on a *single-resolution model* are reported in [5].

In this paper we resort to a recently developed modeling paradigm, based on the specification of Markov models on hierarchical graph structures [3, 8, 9], in order to provide optimal (in the bayesian sense) non-linear algorithms for multiresolution data fusion. The remainder of this paper is organized as follows.

In section 2, we introduce the hierarchical Markov models that will be used to represent the images in the data fusion process. The different hierarchical graph structures (quadtree and ternary graph), as well as the underlying statistical assumptions are presented in detail. Section 3 is devoted to the derivation of optimal bayesian estimators, based on the Markov models defined in section 2 and associated with several different cost functions. A first algorithm, inspired by Bouman's [3] work on single-resolution data processing using multiscale models, is described. An original (Viterbi-like) algorithm is then introduced. Experimental results are reported in section 4, in the special case of multiresolution data segmentation. The non-iterative algorithms are compared on synthetic and real-world data, from a qualitative and quantitative point of view.

2 Hierarchical statistical models

2.1 A hierarchical estimation structure

We consider the standard problem of estimating a hidden label field X from an observation set Y according to some Bayesian criterion. More precisely, X is

a random process indexed by the nodes of a hierarchical graph S , like the graph represented in figure 2 (quadtree) or the structure of figure 3 (called "ternary graph" since a node has generally three "fathers"). The nodes of the graph which belong to the same resolution level form a set S^n , where n denotes the scale. Scale 0 corresponds to the finest resolution whereas scale L stands for the coarsest resolution (figure 1).

The restriction of X to S^n is denoted by $X^n \triangleq \{X_s, s \in S^n\}$, where X_s represents the label at node s .

The set of "fathers" of site s is denoted by ∂s . In the case of the quadtree, there is a unique father which is denoted by $s\bar{\gamma}$. For the ternary graph the three fathers are denoted $s\bar{\gamma}, s\bar{\alpha}, s\bar{\beta}$ (see figure 3). Let $\mathcal{D}(t)$ denote the set of children nodes of site s : $\mathcal{D}(t) \triangleq \{s : t \in \partial s\}$. Set $> s$ represents all the "offsprings" of s or, otherwise said, the strict future of s . Likewise $< s$ denotes the set of ancestries of s (see figure 2).

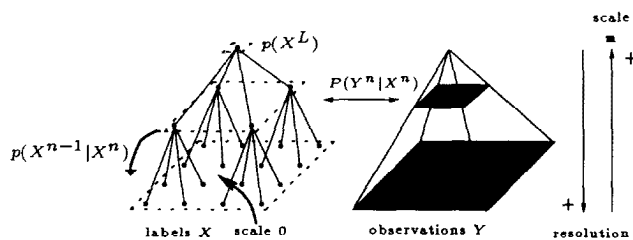


Figure 1: The hierarchical estimation structure

The observations Y are defined on the same graph structure as X (and Y^n denotes the restriction of Y to scale n). Let us notice here that Y_s does not necessarily exist (since data may not be available at all resolutions). Besides, Y_s may correspond to an observation *vector*, in case multispectral observations are available at scale n (if $s \in S^n$).

2.2 Assumptions on the statistical model on the quadtree

Consider the process $X = (X^n)_{n=0 \dots L}$ with $X^n = (X_s)_{s \in S^n}$ and X_s taking its values in Λ (we assume here that Λ is a discrete state space but the extension to a continuous state space is straightforward). In the case of multiresolution data segmentation, for instance, X_s takes its values in $\Lambda = \{1, \dots, M\}$, where M is the number of regions in the scene. Following [3, 9] some simplifications are introduced on the statistical relationship between the different variables of processes X and Y .

- The fundamental assumption consists in considering X as a Markov process in scale n , i.e.:

$$P(X^n | X^i, i > n) = P(X^n | X^{n+1}).$$

Besides, we also assume that the distribution of X_r (where r designates the root) is uniform.

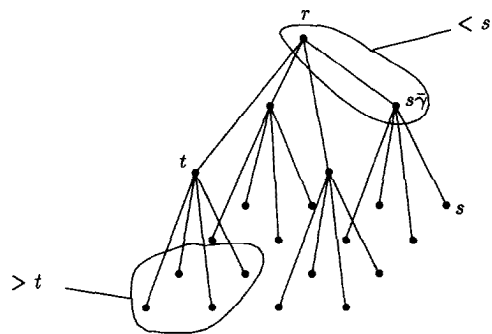


Figure 2: The quadtree graph structure

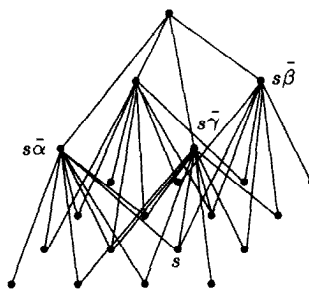


Figure 3: The ternary graph structure

- The scale to scale transitions are assumed to decompose as follows :

$$P(X^n | X^{n+1}) = \prod_{s \in S^n} g^n(X_s | X_{s\bar{\gamma}}).$$

- We also assume that

$$P(Y|X) = \prod_n P(Y^n | X^n) = \prod_n \prod_{s \in S} f^n(Y_s | X_s)$$

(Y is said to be "semi-markovian" with respect to X .)

When site s at scale n supports no observation we take $f^n(y_s | \cdot) \equiv 1$.

With these different assumptions, process $Z = (X, Y)$ is markovian on the quadtree. As a consequence, in the following we will use extensively the standard Markov factorization property on the quadtree ¹ :

$$\forall A, B, C \subset S, \text{ if } B \text{ separates } A \text{ and } C, \text{ then} \\ P(Z_A, Z_C | Z_B) = P(Z_A | Z_B) P(Z_C | Z_B)$$

2.3 Limitations of the quadtree model

In practice, as will be seen in the experimental results, the quadtree structure (as also observed in [3, 9])

¹ B separates A and C iff any chain joining some site of A with some site of C goes through B .

yields “blocky” segmentations. This phenomenon can be explained by the fact that two neighbouring sites, at a given scale, may not have the same father. In this case, a boundary appears far more easily than if they were linked by a parent node. In order to avoid these blocky artefacts, we can use more connected graph structures such as the “ternary-graph” (presented figure 3), in which all neighbouring sites, at a given scale, have at least a common parent node. In this case the estimated field become smoother and block artefacts disappear. The statistical assumptions are essentially the same in this case and are not presented here for the sake of concision.

3 Bayesian Estimation and Algorithms

We consider the general bayesian estimation problem which consists in minimizing the expectation of a cost function C given the observations $Y = y$:

$$\hat{x} = \arg \min_x \mathbb{E}(C(X, x)|Y = y) \quad (1)$$

where X is the random process which stands for the “true” solution and $C(X, x)$ is the cost we have to pay if we select x instead of the true solution X .

A standard cost function is the one that yields the MAP (*maximum a posteriori*) estimation, namely

$$C_{MAP}(X, x) = 1 - \delta(X - x) \quad (2)$$

$$= 1 - \prod_{n=0}^L \delta(X^n - x^n) \quad (3)$$

Unfortunately this criterion is not adapted to a hierarchical structure such as the quadtree. Indeed, if we look at equation (3), we see that this function penalizes mislabeled sites independently of their corresponding scales. In other words, an error at a coarse scale will be paid the same price as an error at a finer scale. In fact one should pay a higher price for errors at coarse levels because these errors correspond to large regions at finer scales.

An example of cost function which penalizes errors according to the scale is the SMAP (S is for Sequential) cost function introduced by Bouman in [3] :

$$C_{SMAP}(X, x) \triangleq \sum_{n=0}^L 2^n C_n(X, x),$$

where

$$C_n(X, x) \triangleq 1 - \prod_{i=n}^L \delta(X^i - x^i).$$

As can be observed, with this cost function a mislabeling at scale $n + 1$ will cost twice the price of an error at scale n . We present in the next paragraph an algorithm inspired from Bouman’s algorithm but adapted to the case of multiresolution data fusion.

3.1 Modified Bouman Algorithm

We extend here the algorithm proposed by Bouman to take into account multiple (multiresolution) observations.

3.1.1 The quadtree model

We address the following estimation problem on the quadtree :

$$\begin{aligned} & \min_x \mathbb{E}_X \left\{ \sum_{n=0}^L 2^n C_n(X, x) | Y = y \right\} \\ &= \max_x \sum_{n=0}^L 2^n \mathbb{E}_X \left(\prod_{i=n}^L \delta(X^i - x^i) | Y = y \right) \\ &= \max_x \sum_{n=0}^L 2^n P(X^i = x^i, i \geq n | Y = y) \end{aligned}$$

Using the markovianity of (X, Y) , we write

$$P(x^n | y, \hat{x}^{n+1}) = P(x^n | y^l, l \leq n, \hat{x}^{n+1}).$$

Following Bouman [3], the following estimation formulae are derived :

$$\hat{x}^L \simeq \arg \max_{x^L} P(y | x^L) P(x^L)$$

$$\hat{x}^n \simeq \arg \max_{x^n} P(y^l, l \leq n | x^n) P(x^n | X^{n+1} = \hat{x}^{n+1}). \quad (4)$$

Since (X, Y) is markovian it comes :

$$\begin{aligned} P(y^l, l \leq n | x^n) &= \prod_{s \in S^n} P(y_{>s} | x_s) \\ &= \begin{cases} \prod_{s \in S^0} f^0(y_s | x_s) & \text{if } n = 0 \\ \prod_{s \in S^n} f^n(y_s | x_s) P(y_{>s} | x_s) & \text{if } n > 0 \end{cases} \end{aligned}$$

and :

$$\begin{aligned} P(y_{>s} | x_s) &= \sum_{x_{\mathcal{D}(s)}} P(y_{>s}, x_{\mathcal{D}(s)} | x_s) \\ &= \sum_{x_{\mathcal{D}(s)}} \prod_{t \in \mathcal{D}(s)} P(y_{>t} | x_t) P(x_t | x_s) \end{aligned}$$

$$= \begin{cases} \prod_{t \in \mathcal{D}(s)} \sum_{x_t} f^0(y_t | x_t) g^0(x_t | x_s) & \text{if } s \in S^1 \\ \prod_{t \in \mathcal{D}(s)} \sum_{x_t} f^{n-1}(y_t | x_t) g^{n-1}(x_t | x_s) P(y_{>t} | x_t). \end{cases}$$

Let us now define

$$h_s^n(\lambda) \triangleq P(y_{>s} | X_s = \lambda) \quad n \geq 1 \text{ and } \lambda \in \Lambda.$$

$h_s^n(\lambda)$ may be computed using the following recursive fine-to-coarse computations :

$$h_s^1(\lambda) = \prod_{t \in \mathcal{D}(s)} \sum_{\lambda' \in \Lambda} f^0(y_t | \lambda') g^0(\lambda' | \lambda)$$

$$h_s^n(\lambda) = \prod_{t \in \mathcal{D}(s)} \sum_{\lambda' \in \Lambda} f^{n-1}(y_t | \lambda') g^{n-1}(\lambda' | \lambda) h_t^{n-1}(\lambda'). \quad (5)$$

Finally, assuming that the distribution $X^L = X_r$ is uniform and using the markovian properties of (X, Y) , the SMAP solution is given by the following coarse-to-fine recursive computations :

$$\begin{aligned}\hat{x}_r &= \arg \max_{k \in \Lambda} h_r^L(k) f^L(y_r | k), \text{ where } r \text{ is the root} \\ \hat{x}_s &= \arg \max_{k \in \Lambda} h_s^n(k) f^n(y_s | k) g^n(k | \hat{x}_{\bar{s}}), 0 < n < L \quad (6) \\ \hat{x}_s &= \arg \max_{k \in \Lambda} f^0(y_s | k) g^0(k | \hat{x}_{\bar{s}}), n = 0.\end{aligned}$$

Thus, the global optimization problem (1) has been transformed in a multitude of local problems which can be handled simultaneously. This is also the case for the Viterbi-like algorithm presented in the next section. The algorithm includes two steps : the first one consists in computing the functions $h_s^n(\cdot)$ for all the sites from the leaves to the root of the tree and the second one consists in a coarse-to-fine label estimation by solving at each node Eqn. (6).

In the above expressions appear two kinds of terms (like in energy functions derived from a Markov Random Field (MRF) model). The first one expresses the relationship between observations and labels, while the second one stands for the interactions between neighbouring labels on the tree (*i.e.*, between a node and its parent node). The latter term is similar to the clique potentials defined in MRF models and provides a contextual *a priori* information. However, unlike MRF models, where the neighbourhood system is defined over a single scale, the hierarchical model introduced here induces both interactions between scales, and interactions between points at a given scale *via* their parent nodes. Besides defining natural interactions between multiresolution data, this kind of structure also supports *causal* processes that (unlike MRF) yield non-iterative algorithms.

3.1.2 Extension to the “ternary graph”

As already explained, the quadtree graph structure induces blocky artefacts on the estimated process. The ternary graph does not yield this kind of artefact, thanks to the additional links introduced between nodes (see figure 3). Unfortunately the computation of pseudo-likelihood functions with recursive equations like (5) is not possible any more due to the form of the transition probability function (10). The solution proposed by Bouman consists in considering an “hybrid graph” which evolves dynamically. More precisely, one first considers the quadtree model and uses the recursive equations (5) for the fine-to-coarse step, and then introduces the ternary graph model for the coarse-to-fine step. Although this yields a solution that is not exact, it leads to satisfactory results in practice, as will be seen in section 4.

3.2 Viterbi-like Algorithm

We introduce here an original algorithm on the quadtree related to the standard Viterbi algorithm. This one is an *exact* MAP estimation on the quadtree.

3.2.1 MAP cost function

The MAP estimator is defined as :

$$\hat{x} \triangleq \arg \max_x P(x | y) = \arg \max_x P(x, y).$$

We have :

$$\begin{aligned}\max_x P(x, y) &= \max_{x_{\geq r}} P(x_{\geq r}, y_{\geq r}) \quad (7) \\ &= \max_{x_r} \left\{ P(x_r, y_r) \max_{x_{>r}} P(x_{>r}, y_{>r} | x_r) \right\} \\ &= P(\hat{x}_r, y_r) \prod_{s \in \mathcal{D}(\tau)} \max_{x_{\geq s}} P(x_{\geq s}, y_{\geq s} | \hat{x}_r)\end{aligned}$$

where \hat{x}_r is the label that reaches the maximum in (7). Now, let us define

$$P_s^*(x_{\bar{s}}) \triangleq \max_{x_{\geq s}} P(x_{\geq s}, y_{\geq s} | x_{\bar{s}})$$

The following recursive relation is thus derived :

$$P_s^*(x_{\bar{s}}) = \max_{x_s} \left\{ P(x_s, y_s | x_{\bar{s}}) \prod_{t \in \mathcal{D}(s)} P_t^*(x_s) \right\} \quad (8)$$

Like in the modified Bouman algorithm, the MAP Viterbi algorithm consists in a fine-to-coarse step followed by a coarse-to-fine one :

- The first step consists in computing P_s^* as a function of $x_{\bar{s}}$ (we stop just before the root), *i.e.*
 - on S^0 : $P_s^*(x_{\bar{s}}) \triangleq \max_{x_s} f^0(y_s | x_s) g^0(x_s | x_{\bar{s}})$
 - on $S^n, n < L$: P_s^* is computed from $P_t^*, t \in S^{n-1}$ according to the recursive equation (8).

Simultaneously we store the $\hat{x}_s(x_{\bar{s}})$ reaching the maximum of P_s^* .

- The second step starts at the root of the tree, (assuming X_r has uniform distribution) and goes down to the leaves as follows :

$$\begin{aligned}- \hat{x}_r &= \arg \max_{x_r} \left\{ f^L(y_r | x_r) \prod_{s \in \mathcal{D}(\tau)} P_s^*(x_r) \right\} \\ - \text{for } s \in S^n, 0 \leq n < L, \hat{x}_s &\text{ is the label which reaches } P_s^*(\hat{x}_{\bar{s}}).\end{aligned}$$

4 Experimental results

4.1 Application to multiresolution image data segmentation

We consider the problem of partitioning a scene using multiresolution image data. Labels X_s take their

values in $\Lambda = \{1, \dots, M\}$ which represent region labels (the number of classes M is assumed to be known "a priori"). For this case study, we consider gaussian observations :

$$f^n(y_s | X_s = k) \triangleq \frac{e^{-1/2(y_s^n - \mu_k)^T \Gamma_k^{-1} (y_s^n - \mu_k)}}{(2\pi)^{d/2} \det(\Gamma_k)^{1/2}}$$

where d is the number of images at scale n , while μ_k is the mean of class k and Γ_k is the covariance matrix (which are estimated by supervised methods).

Following [3], for the quadtree model, we take ²

$$g^n(m|k) \triangleq \begin{cases} \theta_n & \text{if } m = k \\ \frac{1-\theta_n}{M-1} & \text{otherwise} \end{cases} \quad (9)$$

and for the "ternary graph"

$$\tilde{g}^n(m|i, j, k) \triangleq \frac{\theta_n}{7} (3\delta_{m,i} + 2\delta_{m,j} + 2\delta_{m,k}) + \frac{1-\theta_n}{M} \quad (10)$$

where θ_n is a parameter and k is the parent node of the node of S^n labeled m . In the case of the ternary graph i, j, k are the labels of the parent nodes of label m . Parameters θ_n are estimated by an EM-like algorithm [1].

Two algorithms are compared here: the modified Bouman algorithm on the ternary graph and the MAP Viterbi-like algorithm on the quadtree. The rates of correct classification, as well as the cpu time (on a SUN Sparc Workstation) are given for the synthetic images.

4.2 Synthetic images

The synthetic images (figure 4) show circular-shaped regions of different sizes and grey levels standing out against a homogeneous background. This synthetic example is close to the one presented by Bouman in [3], thus the results may be compared.

A single or two resolution levels have been used in turn, in order to show the contribution of data fusion. The data have been corrupted by additive Gaussian white noise. The resolution of the different image data, the signal-to-noise ratio as well as the percentage of correct classification are presented in table 1. As can be seen, the best results, for high signal-to-noise ratios are obtained for the modified Bouman algorithm. The Viterbi (MAP) algorithm seems however to be more robust to high noise levels. In any case, the fusion of two data fields, using the hierarchical Markov model, increases the classification rate in this method. The enhancement is particularly significant for high noise levels.

4.3 Application to multiresolution aerial images

We have applied the same algorithms to aerial images (in the visual spectrum) of the area of Saint-Louis. They were taken during the historical flooding

²To simplify the notations, we write $g^n(m|k)$ for $g^n(X_s^n = m | X_s^{n+1} = k)$ and $\tilde{g}^n(m|i, j, k)$ for $\tilde{g}^n(X_s^n = m | X_s^{n+1} = i, X_s^{n+1} = j, X_s^{n+1} = k)$

ALGO	Image Sizes	SNR dB	%correct classif.	Time
MAP	256 × 256 64 × 64	-16 1	82	26s
SMAP	256 × 256 64 × 64	-16 1	93	49s
MAP	256 × 256	-16	73	26s
SMAP	256 × 256	-16	35	36s
MAP	256 × 256 64 × 64	-10 1	96.1	26s
SMAP	256 × 256 64 × 64	-10 1	97.5	40s
MAP	256 × 256	-10	96.0	26s
SMAP	256 × 256	-10	97.4	36s

Table 1: Results on synthetic images. Time: cpu time on a Sun Sparc 10.

of Mississippi and Missouri rivers in July 1993. We have considered 4 classes : the Mississippi river, the Missouri river, urban area (roads, buildings ...) and vegetation area. The parameters of classes (means and variances) have been estimated by supervised methods (learning on a training set). These results can be compared to those obtained in [11] by standard multispectral methods.

Figure 5 shows the data and label structures used in the algorithms. Data were defined over two grids of different resolutions (namely 512 × 512 and 128 × 128). The label fields are estimated here using the modified Bouman algorithm, which, as can be seen, gradually refines the segmentation by combining more and more multiresolution image data. This example shows the contribution of data fusion. As a matter of fact, it is not possible to discriminate the two rivers by taking into account only the finest image. The fusion with a second data field (at a different resolution and in a different spectral domain) allows the discrimination.

5 Conclusion

We have presented two non-iterative algorithms for the fusion of multiresolution images. Let us notice that the mathematical framework is comprehensive and might be applied to other low level vision problems. As for the theoretical continuation of this work, we plan to devise extension of the Viterbi algorithm to ternary-like graph structure. The problem of parameter estimation is also under study. The aim is to derive specific EM-like algorithms which take best advantage of the particular structures we deal with.

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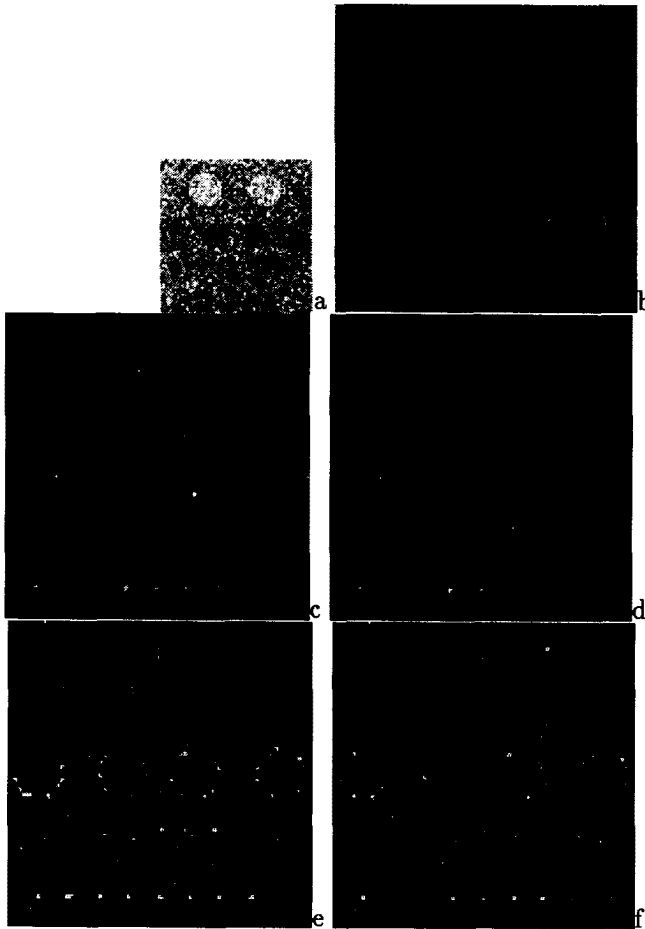


Figure 4: a) 64×64 observation field (standard-deviation of the noise : 30, 1 dB), b) 256×256 observation field (standard-deviation of the noise : 30, -10 dB), c) Result of the modified Bouman algorithm with fusion of images a and b, d) Result of the modified Bouman algorithm with image b, e) Result of the Viterbi algorithm (MAP) with images a and b, f) Result of the Viterbi algorithm (MAP) with image b.

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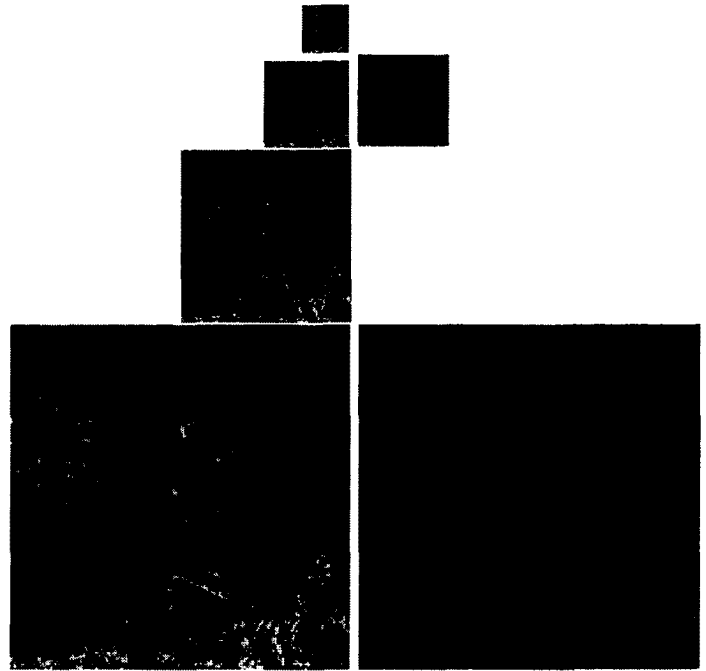


Figure 5: Result of the modified Bouman algorithm for the “Mississippi” images. Images are on the right, results on the left (each class is represented by one color). CPU time 1mn40s on a Sun SPARC 10.

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