Algorithms for Construction of Optimal and Almost-Optimal Length-Restricted Codes

(Abstract)

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Given a list of items $e_1, e_2, \ldots, e_n$ with weights $p = p_1, p_2, \ldots, p_n$ respectively, a code $\mathcal{L}$ with codeword lengths $l_1, l_2, \ldots, l_n$ is a *length-restricted prefix-free code,* if no codeword is a prefix of another one and $l_i \leq L$ for some constant L and all i. A code $\mathcal{L}$ is a *minimum redundancy length-restricted code* if $Length(\mathcal{L}, p) = \sum l_i p_i$ is minimal among all length-restricted prefix-free codes. The problem of length-restricted coding is motivated by practical implementations of coding algorithms. See [KN05] for a description of the previous work on this topic.

In this paper we present a parallel algorithm for the construction of minimum-redundancy length-restricted codes that is based on the Package-Merge algorithm of Larmore and Hirschberg [LH90]. Our algorithm constructs a length-restricted code in $O(L)$ time with n processors on a CREW PRAM. Thus our algorithm has the same time-processor product as the sequential algorithm of [LH90]. Our parallelization of the Package-Merge is based on dividing elements into classes W_l, which contain elements in range $[2^{l-1}, 2^l)$. Relative weight $r(t)$ of an element $t \in W_l^i$ is $weight(t) \cdot 2^{-l}$. We maintain for every item $e \in W_l^j$ and every i, $l \leq i \leq l + \log n$ the value of $pred(e, i) = k$, s.t. $S^1[k] \in W_i^1$ and $r(S^1[k]) \leq r(e) < r(S^1[k+1])$.

We also consider the problem of constructing the *almost-optimal* length-restricted codes. A code $\mathcal{L}$ is n almost-optimal code with error ϵ, if $AvLen(\mathcal{L}, \bar{p}) \leq AvLen(\mathcal{L}', \bar{p}) + \epsilon$ for all length-restricted codes $\mathcal{L}'$, where $AvLen(\mathcal{L}, \bar{p}) = Length(\mathcal{L}, \bar{p})/P$ and $P = \sum_{i=1}^{n} p_i$. The idea of our algorithm for almost-optimal length-restricted codes is to replace weights p_i with weights $p_i^{new} = p_i / \lceil P/n^k \rceil$, for some $k > 0$.

Theorem 1 *An optimal length-restricted code with maximum codeword length L can be constructed in $O(L)$ time with n CREW processors. For any $k > 0$, an almost-optimal length-restricted code with error $1/n^k$ can be constructed in $O(kn \log n)$ time, or in $O(k \log n)$ time with n CREW processors.. If $k \leq L/\log_\Phi n$, a length-restricted code with error $1/n^k$ can be constructed in $O(n)$ time or in $O(k \log n)$ time with $n/\log n$ CREW processors.*

A description of the algorithms is provided in the full version of this paper[KN05].

References

[LH90] L. Larmore, D. Hirschberg, *A Fast Algorithm for Optimal Length-Limited Huffman Codes,* Journal of the ACM 37(3) (1990), pp. 464–473.

[KN05] M. Karpinski, Y. Nekrich, *Algorithms for Construction of Optimal and Almost-Optimal Length-Restricted Codes,* full version, Research Report) University of Bonn, 2005; available at `http://theory.cs.uni-bonn.de/Zope/English/`

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