On Wavelet Compression of Self-Similar Processes

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Self-similar stochastic processes are stochastic counterparts of deterministic fractals. Statistical properties of those processes are the same regardless of the resolution in which they are *observed*. Many natural images have parts that closely resemble one another even when viewed at different resolutions. This simple observation has led to thorough investigations of self-similarity properties in image modelling and applications. Numerous papers have dealt with appropriateness and applications of self-similarity and its close counterpart "scale-invariance" in natural and medical image processing.

Fractional Brownian motion (fBm) is a self-similar non-stationary Gaussian process originally proposed to model power-law behavior of power spectrum of long-range dependant (LRD) natural processes. The first attempts to rigourously quantify the information theoretic properties of fBm has just been started.

Multi-scale nature of wavelets make them natural candidates for analysis and synthesis of fractional Brownian motions. Even though the fBm itself is not stationary, its wavelet transform turns out to be stationary at each scale and at synchronous time instants across the scales. This interesting property has been a driving force behind the research on multi-scale analysis of fBm. Despite wavelet compression being the method of choice for image compression, attempts to understand its intrinsic rate-distortion properties were made only recently. A noticeable example of such efforts is the characterization of operational rate-distortion behaviors of wavelet image coders for compression of certain classes of sources, namely piecewise smooth signals in [1].

We have investigated the performance of wavelet compression schemes for compressing fractional Brownian motions.Theoretical rate-distortion function of fBm is explicitly derived in the following theorem:

Theorem .1:

$$D_{fBm} \geq bR_{fBm}^{-2H}$$

for some constant $b > 0$.
The operational rate-distortion behavior of the wavelet coder is shown to be asymptotically the same as the theoretical rate-distortion performance:

Theorem .2:

$$D_{wlt} \leq A_H R_{wlt}^{-2H}$$

for some constant A_H depending possibly on H and the mother Wavelet.

References

[1] A. Cohen, I. Daubechies, O.G. Guleryuz, M.T. Orchard, "On the Importance of Combining Wavelet-Based Non-Linear Approximation with Coding Strategies", IEEE Trans. on IT,VOL 48, July 2002.

[2] V.P. Roychowdhury, J. Gao, *Lomg range dependent processes*,in prepration.

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