Slepian-Wolf Coding of Multiple M-ary Sources Using LDPC Codes

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We consider Slepian-Wolf coding of n correlated m-ary sources X_l's for $l \in \{1, 2, ..., n\}$. Applying the syndrome concept, we show how multi-level codes with low-density parity-check (LDPC) codes at each level can be used to approach the Slepian-Wolf limit.

Let the rate for X_l be R_l. When compressing correlated sources X_l's separately without loss if a joint decoder is employed, the Slepian-Wolf theorem defines the set of achievable rate vectors as [1, p.415]: $R(\mathbf{S}) > H(X(\mathbf{S})|X(\mathbf{S}^c))$, where for all $\mathbf{S} \subseteq \{1, 2, ..., n\}$, $R(\mathbf{S}) = \sum_{l \in \mathbf{S}} R_l$ and $X(\mathbf{S}) = \{X_l : l \in \mathbf{S}\}$. The advantage of LDPC codes is that they can be designed for different correlation models between source outputs and the side information and approach the Slepian-Wolf limits. Specifically, for Slepian-Wolf coding of three sources, a design rule of rates for coding each source was proposed in [2]. This rule not only facilitates code design but also allows multi-stage decoding. The approach in [2] can be extended to n m-ary sources. In the case of compression of n binary sources, ($H(X_1)$, $H(X_2|X_1)$,. . .,$H(X_n|X_1, X_2, \ldots, X_{n-1})$) is a corner point of the region of achievable rates, for which $\sum_{l=1}^{n} H(X_l|X_1, \ldots, X_{l-1}) = H(X_1, \ldots, X_n)$ from the chain rule for the joint entropy. In addition, when $m = 2^k$, we can achieve the corner point through multi-level coding where a binary LDPC code can be designed for the equivalent binary input multi-level output channel at each level.

We start with four binary sources (i.e., $n = 4$ and $m = 2$) and design codes with rate R_4 close to $H(X_4|X_1, X_2, X_3)$, assuming codes of rates R_1, R_2 and R_3 are already available for approaching the limits $H(X_1)$, $H(X_4|X_1, X_2)$ and $H(X_4|X_1, X_2, X_3)$, respectively. For a specific symmetric correlation model that is an extension of that in [2], the simulated performance of the LDPC code of length 5×10^5 designed with this approach is 0.0308 bit away from $H(X_4|X_1, X_2, X_3)$ and is 0.0262 bit lower than $H(X_4|X_1, X_2)$.

In the case of compression of two ternary sources (i.e., $n = 2$ and $m = 3$), a ternary LDPC code is carefully designed with EXIT-charts method based on Davey and MacKay's decoding algorithm. Given a symmetric correlation model specified by one row of the transition matrix [1-2q q q], the rate of the ternary LDPC code designed for $q = 0.1$ is 0.3846 while the capacity is 0.4183. The simulated performance of the length 5×10^5 ternary code is 4.43×10^{-4} (symbol error rate) at $q = 0.085$, for which the capacity is 0.4778. The loss is about 0.0932 in channel coding rate (or $0.0932 \log_2 3 = 0.1477$ bit in compression performance). The larger loss in this ternary case agrees with what has been observed for the AWGN channel, i.e., it is harder to achieve the capacity with non-binary signaling/coding.

For a longer version of this paper, please refer to our web page at http://lena.tamu.edu.

References

[1] T. M. Cover and J. A. Thomas, "Elements of Information Thoery," John Wiley & Sons, 1991.

[2] A.D. Liveris, C.-F Lan, K. R. Narayanan, Z. Xiong and C.N. Georghiades, "Slepian-Wolf Coding of Three Binary Sources Using LDPC Codes," *Proc. 3rd Intl. Symp. on Turbo codes,* , pp. 63-66, Brest, France, September 2003.

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