Steady State Calculation of Oscillators Using Continuation Methods

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Abstract

Shooting, finite difference or Harmonic Balance techniques in conjunction with Newton's method are widely employed for the numerical calculation of limit cycles of oscillators. The resulting set of nonlinear equations are normally solved by damped Newton's method. In some cases however, divergence occurs when the initial estimate of the solution is not close enough to the exact one. A two-dimensional homotopy method is presented in this paper which overcomes this problem. The resulting linear set of equations employing Newton's method is under-determined and is solved in a least squares sense for which a rigorous mathematical basis can be derived.

1 Limit cycle calculation of oscillators

We consider the autonomous circuit differential algebraic equations (DAEs) of dimension N

$$f(x(t)) = i(x(t)) + \frac{d}{dt}\, q(x(t)) = 0 \tag{1}$$

We assume that

- the DC operating point is unstable, i.e. $i(x_{DC}) = 0$ is an unstable solution of (1) and
- the circuit DAE exhibits a periodic limit cycle $x_{ss}(t) = x_{ss}(t+T)\ \forall\, t$ where the period T is a priori unknown.

The boundary value problem can be solved by finite difference techniques: For a numerical treatment (1) might be discretized using some sort of finite difference schemes choosing K nodes on an equidistant mesh with the grid spacing $\Delta t = \frac{T}{K}$, where T is the a priori unknown period of the limit cycle:

$$F(X) = I(X) + \nabla\, Q(X) = 0. \tag{2}$$

Here $F,\ X,\ I,\ Q$ are vectors of the size $N \cdot K$ and ∇ represents the discretized time derivative. Unfortunately, finite difference techniques often fail to converge when using plain Newton's method.

The homotopy technique applied to (2) sweeps from an initial value problem ($\lambda = 0$) to a periodic boundary value problem ($\lambda = 1$):

$$F(X, \lambda_1, \omega) = I(X) + \hat{\nabla}\, Q(X) + \bar{\nabla}\, Q(\hat{X}) = 0$$
$$\hat{X} = \lambda_1\, X + (1 - \lambda_1)\, X_0, \quad 0 \le \lambda_1 \le 1 \tag{3}$$

where $\hat{\nabla}$ and $\bar{\nabla}$ are block lower triangular and strictly block upper triangular matrices, respectively, and $\nabla = \hat{\nabla} + \bar{\nabla}$. Note that in (3) F is a map $F\ :\ \mathbb{R}^m \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}^m$, where $m = N \cdot K$. Associate now the normalized angular frequency ω/ω_0 with a second homotopy parameter λ_2. ω_0 is an initial estimate. The two-dimensional homotopy method is therefore started with the initial conditions $(\lambda_1, \lambda_2) = (0, 1)$. The homotopy method employed is a generalization of a technique presented by Deuflhard et al [1].

The continuation method has been applied to different oscillators where plain Newton methods failed because the initial estimates of the solution were to far away from the exact one. Most circuits are quartz crystal oscillators where finding the limit cycle is very difficult.

	Quadrature	Clapp	Colpitts	Pierce
f_0/MHz	131	20	20	2
$\frac{f}{f_0}$	0.056	0.999978	0.99996	0.999976
steps	29	16	16	16

Table 1. Results for various oscillators

Table 1 shows initial estimate f_0, final ratio f/f_0 and number of iterations for various oscillators. The number of iterations is acceptable for all circuits under test.

References

[1] P. Deuflhard, B. Fiedler, P. Kunkel. *Efficient Numerical Pathfollowing beyond Critical Points*. Technical Report 278, Universität Heidelberg, 1984.

Proceedings of the 2002 Design, Automation and Test in Europe Conference and Exhibition (DATE'02)
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