

# An Approach to Model Checking for Nonlinear Analog Systems

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## Abstract

*We present the first approach to model checking for nonlinear analog systems. Based on digital CTL model checking ideas, results in hybrid model checking and special needs in analog verification, a new model checking tool has been implemented.*

*Published model checking tools for hybrid systems require discrete or partly linear system descriptions. Our focus is on nonlinear analog behavior, therefore a new approach is necessary. There are mainly two aspects to be considered. Firstly, a discrete model retaining the essential nonlinear analog behavior has to be developed. Secondly, model checking for analog systems requires extensions of the language to define analog system properties in a reasonable way.*

## Discrete Model Generation

Discrete model generation is splitted into three steps: a time discrete point to point relation, state space subdivisions and generation of the connection relation.

Time discrete algorithms are well known in analog simulators, like Spice, Spectre, Saber, etc. During transient simulation the differential equation system is solved by numerical integration using discrete time steps. Given a small time step, the transition between the actual state and the next state is determined by a numerical integrator, e.g. the backward Euler formula. A step size control uses a local measurement of the integration error. An arbitrary test point in the state space is mapped to its successor state, depending on the actual  $\Delta t$  used.

To get a discrete and finite state description, the continuous and infinite state space has to be bounded and subdivided. This is done by rectangular boxes, which are not necessarily the best choice. However, for implementation reasons boxes are the far most convenient data structure. Restriction to a finite region is simply done by a user defined start area, comprising the considered system behavior. Firstly, a uniform subdivision in all state space dimensions is used. Secondly, an automatic subdivision strategy is used

to react on different system dynamics, depending on the actual state space region. The main target is to get a uniform behavior in each state space box. The uniformity is measured by the variation of the point to point relation, calculated in the state space. Namely, the length  $l_m$  and the angle  $a_m$  of this vector are considered. Box subdivision is continued recursively until  $l_m$  and  $a_m$  drop under a given threshold or a given subdivision depth is exceeded.

The last step in getting a discrete system model is the connection relation between state space regions. A number of test points is taken to estimate the target region. Even a few test points may give a reasonable target approximation, but the region is always underestimated. There are several approaches to make this method rigorous which means to safely include the target region. To expand this approximation to the state space subdivision used, a second step is needed. The final target region defines the connection relation for the actual test region.

## Analog CTL Model Checking

Given the subdivided state space and the connection relation, the continuous problem has been transferred into a discrete model. Thereby, it is treatable by the developed model checker which is based on standard CTL algorithms. The language has been expanded by operators to describe half planes in the state space, e.g. ( $I_L > 3.256$ ). In combination with boolean operators this enables the definition of arbitrary Manhattan polytopes. If the threshold value used in the CTL formula is not already an subdivision value in the state space, it has to be added before executing the CTL formula. This makes the discrete model not only depending on the analog system but also on the CTL formula used.

The prototype has been successfully tested with a tunnel diode oscillator circuit, a second order lowpass filter including a nonlinear opamp model, and a Schmitt trigger circuit. As far as we know, this tool is the first approach to model checking for nonlinear analog systems. It opens a wide range of possibilities in applying formal methods not only to digital and hybrid systems but also to analog ones. Thus, it is a step towards a more formalized analog design flow.